CSGB CENTRE FOR **STOCHASTIC GEOMETRY** AND ADVANCED **BIOIMAGING**

Annual Report 2014

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AND ADVANCED **BIOIMAGING**

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INTRODUCTION

Official opening of CSGB 2 November 2010.

In front (from left to right): Peter Landrock (member of the board of the Villum Foundation) and Lars E. Kann-Rasmussen (chairman of the board of the Villum Foundation). Second row (from left to right): Lauritz Holm-Nielsen (rector of Aarhus University) and Erik Meineche Schmidt (dean of Faculty of Science, Aarhus University). The positions mentioned were the ones held in 2010.

The first internal CSGB workshop, 26 – 27 May 2010, Vingstedcentret, was arranged by the stochastic geometry group (AU). A total of nine such workshops have been arranged during the first funding period. At these workshops, the present status of the CSGB research projects are discussed.

A major event in 2011 was the *16th Workshop on Stochastic Geometry, Stereology and Image Analysis*, 5 – 10 June 2011, Sandbjerg. During the first funding period, CSGB researchers have taken part in the organization of more than 40 scientific meetings.

The Centre for Stochastic Geometry and Advanced Bioimaging (CSGB) was founded in 2010 as a VKR Centre of Excellence. Its main aim is to advance the discipline of **stochastic geometry**, and thereby laying the mathematical foundations for novel methods of analyzing advanced **bioimaging** data. Stochastic geometry, a research field at the cutting edge between mathematics and statistics, is concerned with modelling and inference for random geometric structures. An important subfield is stereology by means of which valid information can be obtained on a spatial structure from sections through the structure. **Spatial statistics** provides methods of statistical analysis of stochastic geometry models.

Research topics at CSGB include rotational integral geometry, digital stereology, point processes and random shapes. In the first funding period of CSGB (1 April 2010 – 31 March 2015, CSGB I), important breakthroughs in the mathematical foundations for the analysis of bioimaging data have been made. Statistical inference has been developed for a whole range of stochastic geometry models. New quantitative bioimaging techniques have been developed by fertile cross-disciplinary research collaborations, including techniques for analysis of molecular microscopy data. Since the foundation of CSGB in 2010, more than **120 scientific papers** have been published by CSGB researchers, including a number of papers in high impact scientific journals. Through this scientific work, it has been possible to develop new mathematical and statistical methods of modelling different types of bioimaging data. In a broader perspective, a very fruitful environment for **scientific interaction** between the four participating research groups has been created.

On 28 October 2014, CSGB received the message that the Villum Foundation will support CSGB with DKK 30 million for a **second five-year funding period**, starting from 1 April 2015. With this grant, the Foundation wants CSGB to continue and further develop its successful research activities in mathematical and statistical methods of analyzing advanced bioimaging data.

The second funding period of CSGB (CSGB II) will involve an intensified collaboration between the **four research groups** that created CSGB together: the *stochastic geometry group* at Department of Mathematics, Aarhus University (AU), the *stereology and EM research laboratory*, AU, the *spatial statistics group* at Department of Mathematical Sciences, Aalborg University (AAU), and the *image section*, Department of Computer Science, University of Copenhagen (KU). The new research plan for the second funding period (1 April 2015 – 31 March 2020) will highly benefit from the established synergy between the four research groups. New advanced basic research questions in integral and stochastic geometry will be addressed and the challenge of new types of bioimaging data will be taken up. More specifically, we will focus on

- *tensors in stochastic geometry*, including (i) their integral geometry, (ii) their role in stochastic modelling and (iii) their estimation from observations in digital grey-value images
- *modelling and statistical inference in non-Euclidean spaces*, including (i) point processes on linear networks and directed graphs, (ii) deformation modelling and statistics of deformations and (iii) models for random trees
- a collection of new projects involving *analysis of concrete bioimaging data* obtained by diffusion weighted imaging, scanning electron microscopy and super-resolution fluorescence imaging

Since 2010, CSGB researchers have taken part in the organization of more than 40 scientific meetings, including **20 international PhD courses**. We will continue this outreach mission in the second funding period. The recognition of CSGB, obtained nationally as well as internationally during the first funding period, will in the second funding period be used as driving force in the formation of new formalized collaborations with major international research groups. The close collaboration with the spatial statistics group in Perth and the stochastic geometry group in Karlsruhe will be continued.

The new grant represents a unique possibility for continuing and further develop these activities.

March 2015, Eva B. Vedel Jensen

The *Workshop on Geometry and Statistics in Bioimaging: Manifolds and Stratified Spaces*, 8 – 12 October 2012, Sandbjerg, was arranged jointly by the stochastic geometry group (AU) and the image group (KU). This workshop has resulted in a number of spin-off activities.

In 2013, the *Summerschool on Topics in Space-Time Modelling and Inference* was arranged by the spatial statistics group (AAU), 27 – 31 May, at Aalborg University. More than 20 international PhD courses have been organized since the etablishment of CSGB in 2010.

The Workshop on Tensor Valuations in Stochastic Geometry and Imaging, 21 – 26 September 2014, Sandbjerg, dealt with one of the three major research topics that will be taken up in the second funding period of CSGB. For further details, see text.

www.csgb.dk

ORGANIZATION AND STAFF

ORGANIZATION AND STAFF

STAFF

CSGB comprises the following four internationally recognized Danish research groups with unique complementary competences in stochastic geometry and bioimaging:

AU-math: *the stochastic geometry group at Department of Mathematics, AU*
Competences in stereology, stochastic geometry, topology

AU-bio: *the stereology and EM research laboratory, AU*
Competences in light microscopy, fluorescence microscopy, electron microscopy

AAU: *the spatial statistics group at Department of Mathematical Sciences, AAU*
Competences in spatial point processes, spatio-temporal point processes, computational statistics

KU: *the image section at Department of Computer Science, KU*
Competences in stochastic shape modelling, Bayesian inference, partial differential equations for imaging

During the first funding period of CSGB, five researchers have obtained **permanent associate professorships**.

In 2014, the CSGB staff consisted of 7 professors, 14 associate professors, 9 postdocs and 10 PhD students, see page 52-53 for details.

ORGANIZATION

During the first funding period of CSGB, the research of CSGB has been organized along three streams, coordinated by senior researchers at the Centre. Each stream has contained three research projects; the principal investigator(s) of each research project is (are) indicated in the diagram below.

Integral geometry and advanced stereology Eva B. Vedel Jensen Markus Kiderlen	**Statistics of stochastic geometry models** Jesper Møller Jens Ledet Jensen	**Advanced bioimaging** Jens R. Nyengaard Mads Nielsen
Rotational integral geometry Eva B. Vedel Jensen	**Spatial and spatio-temporal point processes** Jesper Møller Rasmus P. Waagepetersen	**Non-uniform sampling** Ute Hahn Jens R. Nyengaard
Digital stereology Markus Kiderlen	**Space-time lattice data** Mads Nielsen Kristjana Jónsdóttir	**Fluorescence microscopy taken to the molecular level** Jens R. Nyengaard Rasmus P. Waagepetersen
Topological properties Andrew du Plessis	**Random shapes** François Lauze Mads Nielsen	**Molecular cryo-EM** Monika Golas, François Lauze, Björn Sander

NEWS ABOUT STAFF

A total of **11 PhD students** have been (partially) funded by CSGB in the first funding period. Below, these PhD students are listed, according to the year in which they have defended their thesis/will hand in their thesis.

2013

Ólöf Thórisdóttir (AU-math)
Advanced local stereology

2014

Mohammad Ghorbani (AAU)
Spatial and spatio-temporal point processes

2015

Katrine Hommelhoff Jensen (KU)
Reconstruction algorithms in cryo-EM

Jay Rai (AU-bio)
Molecular studies of multi-protein macromolecular assemblies using cross-linking approaches

Ina Trolle Andersen (AU-bio/math)
Non-uniform sampling in microscopy

Jan-Otto Hooghoudt (AAU)
Point processes and protein interactions

Mahdieh Khanmohammadi (KU)
Vesicle segmentation and modelling in electron microscopy images

Ali Rafati Hoseinpoor (AU-bio)
Stereological estimation of particle shape and orientation

Farzaneh Safavimanesh (AAU)
Spatial point process modelling for solving the minicolumn hypothesis in neuroscience

2016

Astrid Kousholt (AU-math)
Tensor valuations in stochastic geometry and stereology

Sabrina Tang Christensen (AU-Math)
Digital topology

RESEARCH PLAN FOR CSGB II

The research plan for the second funding period is presented on the next page. The initials of the principal investigators of each work package (WP) are indicated, see also the overview of the scientific staff on page 52-53. Visions for the second funding period are described on page 19-21. Below, we explain the changes made compared to the research plan of CSGB I.

Four of the nine projects from CSGB I have been carried over to CSGB II in an extended and further developed form, taking into account and building on the progress that has been made. The resulting work packages (WPs) are

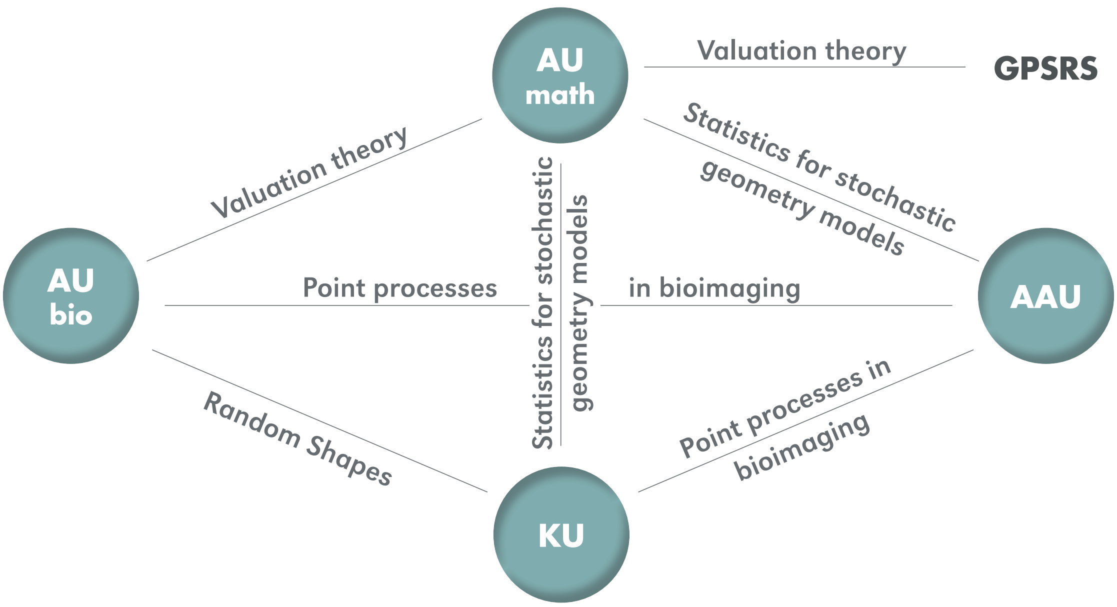
- **Valuation theory** (developed from Rotational integral geometry in CSGB I) – **WP1**
- **Random shapes** – **WP2**
- **Spatial and spatio-temporal point processes** – **WP3**
- **Algorithms** (developed from Digital stereology in CSGB I) – **WP6**

These work packages are extended considerably in scope compared to CSGB I. In addition, the research plan of CSGB II includes the following work packages

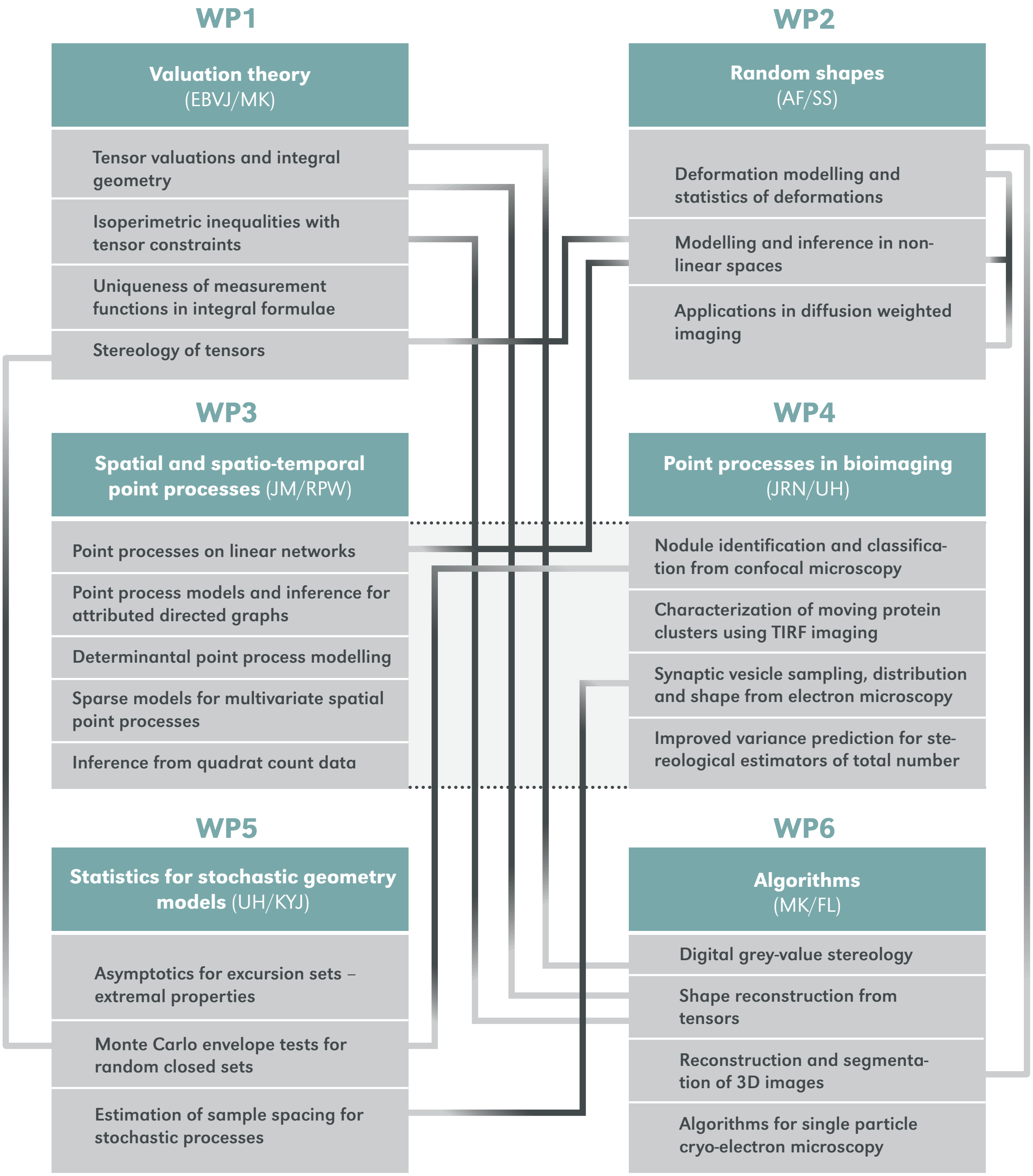
- **Point processes in bioimaging** – **WP4**
- **Statistics for stochastic geometry models** – **WP5**

WP4 concerns new bioimaging applications of stochastic geometry models while **WP5** takes up new statistical inference issues that are directly related to the analysis of bioimaging data. The research plan involves new imaging modalities, viz. diffusion weighted imaging, focused ion beam scanning electron microscopy and super-resolution fluorescence imaging.

A number of the planned projects will strongly benefit from the collaboration of at least two of the participating research groups. The collaboration structure is illustrated below. The work package **Valuation theory** will benefit from a close collaboration with the DFG funded Research Unit entitled *Geometry and Physics of Spatial Random Systems* (GPSRS).



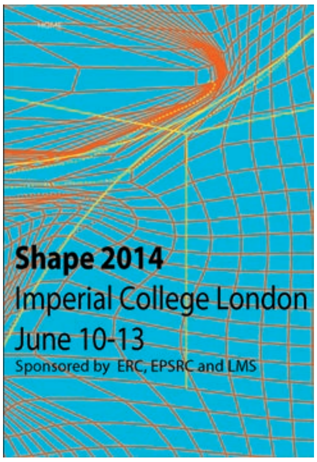
RESEARCH PLAN FOR CSGB II



The leadership of CSGB will be continued in the second funding period (1 April 2015 – 31 March 2020). The director of CSGB II will henceforth be Eva B. Vedel Jensen (EBVJ). The organization of CSGB in the second funding period will otherwise involve some generational changes. Most importantly, Markus Kiderlen (MK) will take over the direction of the stochastic geometry group (AU-math) in Aarhus. He is the ideal candidate for this position, being educated in the stochastic geometry group in Karlsruhe and having a very strong research profile at the interface of geometry on the one side and random set theory and stereology on the other side. The directors of the other research groups will be Jens R. Nyengaard (AU-bio), Jesper Møller (AAU) and Mads Nielsen (KU), as in the first funding period.

INTERNATIONAL EVENTS

In 2014, the image section has participated in the organization of a number of international events, including



Shape 2014 Workshop

10 - 13 June

Imperial College London

Organized by Martins Bruveris (Brunel), Nicolas Charon (KU/Johns Hopkins), Darryl Holm (Imperial College), Henry Jacobs (Imperial College) and Stefan Sommer (KU).

The workshop on shape analysis and deformation modelling attracted among others the creators of the large deformation diffeomorphic metric mapping (LDDMM) framework. Presentations were mixed with highly active discussions on the links between shape analysis, stochastic analysis, deformation modelling and fluid dynamics.

Webpage: https://shape2014.wordpress.com/

Oberwolfach Mini-Workshop - Asymptotic Statistics on Stratified Spaces

28 September - 4 October 2014

Photo: Archives of the Mathematisches Forschunginstitut Oberwolfach.

Organized by Aasa Feragen (KU), Stephan Huckemann (Göttingen), Steve Marron (UNC) and Ezra Miller (Duke).

This was a highly interactive (and fun!) workshop on statistics in stratified spaces and related problems on geometry and statistics. The workshop had 17 participants, with a good mix of junior and senior researchers from geometry, statistics, optimization and image analysis.

Webpage: https://www.mfo.de/occasion/1440a/

FEAST 2014 - ICPR Workshop on Features and Structures

A 1-day workshop in conjunction with the International Conference on Pattern Recognition in Stockholm, 24 August 2014

Data analysis with both discrete structure and continuous features.

Organized by Aasa Feragen (KU), Niklas Kasenburg (KU), Marco Loog (TU Delft and KU) and Veronika Cheplygina (TU Delft). 70 registered participants. Second edition will be organized in 2015.

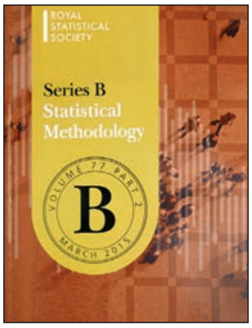
Webpage: https://sites.google.com/site/feast2014/

HIGHLIGHTS 2014

- During 2014, **six papers** by members of the spatial statistics group, AAU, have been published or accepted for publications **in the absolute top international statistical journals**, i.e. *Bernoulli* (1), *Biometrika* (2), *Journal of the American Statistical Society* (1) and *Journal of the Royal Statistical Society B* (2).

- Together with Adrian Baddeley and Rolf Turner, Ege Rubak, AAU, is about to finish the book **Analysing Spatial Point Patterns with R.** This is a practical guide to the analysis of spatial point patterns with amble software illustrations as well as an introduction to the key concepts and models in this field.

- The special issue on **Geometry and Statistics: Manifolds and Stratified Spaces** was published in 2014 as *Journal of Mathematical Imaging and Vision* Vol 50, Issue 1-2.



HONOURS/AWARDs

CADDementia

In conjunction with MICCAI 2014, the image section participated in a challenge of automated diagnosis of Alzheimer's, based on **structural MRI** with no further side information than age available. 29 sets of diagnostic scores on the 354 subjects in the test set were submitted. The **scores** submitted by the image section team from KU (Sørensen-Equal and Sørensen-Optimized) came in as having accuracy **ranked 1st and 2nd**, respectively. The major difference between the DIKU methods and the remaining methods were the capability to analyze the texture of the MRIs. The result of the challenge has now been accepted for NeuroImage:

Bron, E.E., Smits, M., van der Flier, W.M., Vrenken, H., Barkhof, F., Scheltens, P., Papma, J.M., Steketee, R.M., Méndez Orellana, C., Meijboom, R., Pinto, M., Meireles, J.R., Garrett, C., Bastos-Leite, A.J., Abdulkadir, A., Ronneberger, O., Amoroso, N., Bellotti, R., Cárdenas-Peña, D., Álvarez-Meza, A.M., Dolph, C.V., Iftekharuddin, K.M., Eskildsen, S.F., Coupé, P., Fonov, V.S., Franke, K., Gaser, C., Ledig, C., Guerrero, R., Tong, T., Gray, K.R., Moradi, E., Tohka, J., Routier, A., Durrleman, S., Sarica, A., Di Fatta, G., Sensi, F., Chincarini, A., Smith, G.M., Stoyanov, Z.V., Sørensen, L., Nielsen, M., Tangaro, S., Inglese, P., Wachinger, C., Reuter, M., van Swieten, J.C., Niessen, W.J. & Klein, S. (2015): Standardized evaluation of algorithms for computer-aided diagnosis of dementia based on structural MRI: The CADDementia challenge. ***NeuroImage.*** Published online doi: 10.1016/j.neuroimage.2015.01.048.

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RESEARCH

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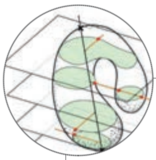

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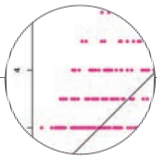

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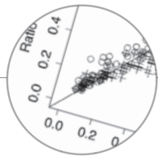

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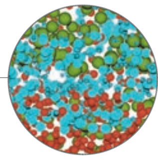

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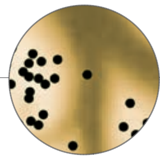

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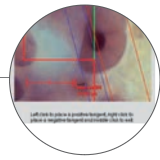

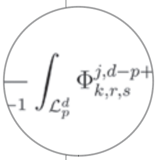

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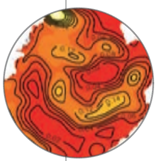

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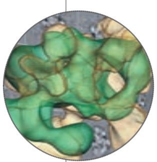

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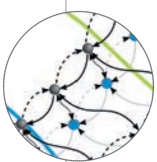

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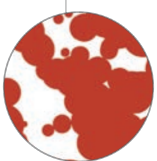

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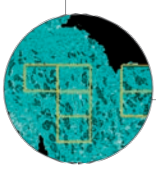
SCIENTIFIC ACHIEVEMENTS

in the first funding period

The scientific output of CSGB in the first funding period is evidenced by the many (more than 120) scientific publications and the high impact journals in which CSGB researchers published, including *Bernoulli, Biometrika, IEEE Transactions on Pattern Analysis and Machine Intelligence, Journal of the American Statistical Association, Journal of Mathematical Imaging and Vision, Journal of the Royal Statistical Society B, Molecular Cell, Nature Neuroscience* and *Statistical Science*.

Since the foundation of CSGB in 2010, central elements of a theory of **rotational integral geometry**, complementary to the theory of translative integral geometry, have been developed for Minkowski tensors. This theory makes it possible to go beyond classical global analysis of a biostructure that often leaves subtle changes in the spatial arrangement of the structure unnoticed. The new theoretical results have been used to develop flexible local methods of describing cell position, cell orientation and cell shape. In **digital stereology**, it was a breakthrough when a second-order asymptotic formula for configuration counts was established. Statistical inference has been developed for a whole range of **point process models**, the fundamental building blocks of many stochastic geometry models. Modelling and statistical analysis in **shape spaces** have been considered in a series of papers. Cornerstones are here numerical methods for non-rigid registration based on mutual information, construction of a metric space of geometric trees and statistical analysis in curved spaces by exact geodesic analysis. Finally, during the first funding period, stochastic geometry models have been introduced for modelling of various microscopy and bioimaging data that give access to the molecular level, including **laser scanning microscopy** and **cryo-electron microscopy (cryo-EM)**.

In a wider perspective, we have in the first funding period succeeded in creating a fruitful environment for mutual scientific interaction between the four involved research groups. An important international event was the **Workshop on Geometry and Statistics in Bioimaging: Manifolds and Stratified Spaces**, 8 – 12 October 2012, Sandbjerg, that was arranged jointly by the stochastic geometry group and the image section. A number of activities have taken place or will take place in the scientific community, as a spin-off of the Sandbjerg workshop.



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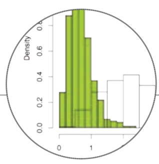

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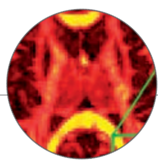

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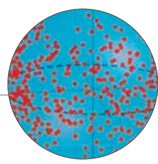

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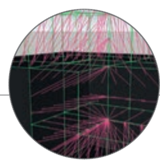

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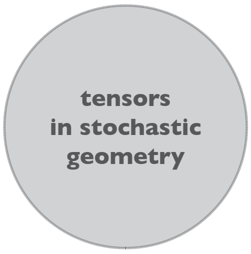

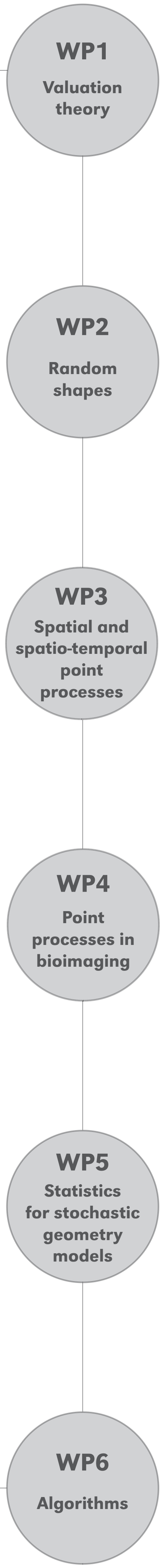

VISION

for the second funding period and beyond

In a second funding period, we will take advantage of the established collaborative structure of CSGB and the increased international collaboration. This enables us to address a number of new challenging basic research questions in integral and stochastic geometry. One focus will be **tensors in stochastic geometry**, including (i) their integral geometry, (ii) their role in stochastic modelling and (iii) their estimation from observations in digital grey-value images. Another important focus point will be **modelling and statistical inference in non-Euclidean spaces**, including (i) point processes on linear networks and directed graphs, (ii) deformation modelling and statistics of deformations and (iii) models for random trees.

At the same time, we will take up a collection of new projects involving analysis of concrete bioimaging data obtained by diffusion weighted imaging, confocal microscopy, super-resolution fluorescence imaging as well as scanning and cryo-electron microscopy. Furthermore, we will follow closely the development of two novel microscopy techniques, **Clarity** (Chung *et al.*, 2013) and **Cubic** (Susaki *et al.*, 2014), and study their implementation. These tissue clearing techniques, that have appeared during the last year, are compatible with various microscopy platforms and open up new opportunities for studying subcellular structures and molecules.

Beyond the second funding period, we will seek funding for establishing international research networks within the themes of CSGB. One obvious possibility is a research network created in close collaboration with our German colleagues in the Research Unit **Geometry and Physics of Spatial Random Systems**.

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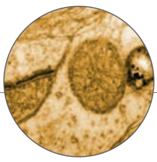
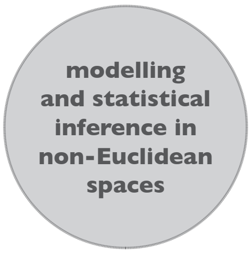
Susaki, E.A., Tainaka, K., Perrin, D., Kishino, F., Tawara, T., Watanabe, T.M., Yokoyama, C., Onoe, H., Eguchi, M., Yamaguchi, S., Abe, T., Kiyonari, H., Shimizu, Y., Miyawaki, A., Yokota, H. & Ueda, H.R. (2014): Whole-brain imaging with single-cell resolution using chemical cocktails and computational analysis. *Cell* **157**, 1–14.





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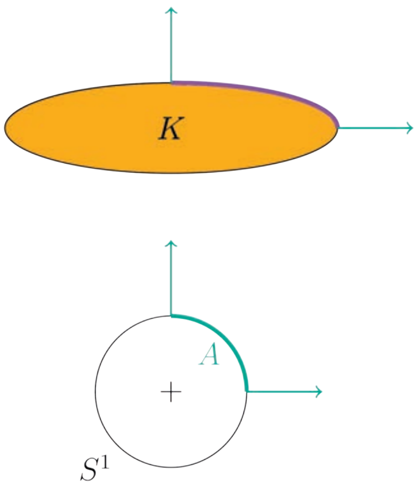

Figure 1:
The area measure $S(K, A)$ of $A \subset S^1$ is the length of the purple part of the boundary of K. The purple points are those that have a unit outer normal in A.

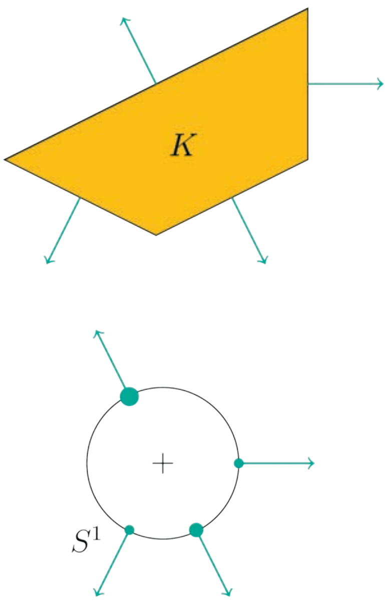

Figure 2:
The area measure of a convex polygon K is a discrete measure. Its support is the set of outer unit normals of the edges of K, and the mass in each of these points is the length of the corresponding edge.

SHAPE RECONSTRUCTION FROM TENSORS

In this section, we will indicate ideas and questions to be addressed in one of the new CSGB II projects, concerning shape reconstruction from tensors.

The classical **shape-from-moments** problem asks how an unknown planar set K can be reconstructed from the moments of the Lebesgue measure on K. Typically, only moments up to a given order are available, and therefore only an approximation of K can be expected.

This reconstruction problem has a large range of applications stretching from the obvious reconstruction of the support of a distribution from its moments to geophysical applications. In computed tomography, the X-rays of an object can be used to estimate moments of the underlying Lebesgue measure on K, allowing to reconstruct the shape of K when the shape-from-moments algorithm is applied. Details can be found e.g. in Elad *et al.* (2004), where K is assumed to be a polygon, and the moments may be corrupted by noise.

In this new CSGB project, we consider the shape-from-moments problem for a planar compact convex set K when moments up to order $s \in \mathbb{N}$ are not taken with respect to the Lebesgue measure on K, but with respect to the area measure $S(K, \cdot)$ of K. (For an illustration of the area measure, see Figure 1.) This corresponds to the knowledge of all surface tensors of K up to rank s. The area measure, which is a finite measure on the unit circle S^1, is determined by all its moments. As K is determined up to translation by $S(K, \cdot)$, see e.g. Schneider (2013), the complete set of moments determines the shape of K, that is, the equivalence class of all translations of K.

If only finitely many moments are available, we can establish a stability result: Assume that K and K' are planar convex sets contained in a ball of fixed radius, with coinciding moments up to order $s > 1$. Then, if δ is the Hausdorff metric, the shapes of K and K' are close to one another in the sense that there exists a translation vector x such that $\delta(K + x, K')$ is of order arbitrarily close to $1/s$. This stability result is based on a generalization of Wirtinger's inequality.

To establish an actual reconstruction algorithm, let $m_s(K)$ be the vector of all moments of $S(K,\cdot)$ up to order $s>0$. This vector has $(s+1)(s+2)/2$ components. If $m_s(K_0)$ is the vector corresponding to an unknown - but sought for - set K_0, we seek an approximation $\widehat{K}_s$ as solution of the least squares problem

$$\min_K \|m_s(K_0) - m_s(K)\|^2, \qquad (1)$$

where the minimum is taken over all convex bodies in the plane. A compactness argument shows that (1) always has a solution. If $m_s(K_0)$ is distorted by independent Gaussian noise, (1) gives the maximum likelihood estimate of K_0. This infinite dimensional problem is intractable numerically, but it can be shown that the set of all solutions contains a convex polygon with at most $2s+1$ vertices. Thus, the optimization in (1) has only to be considered in the finite dimensional space of these polygons, and as the area measure of a polygon is easily related to its geometry (see Figure 2), (1) can be solved using standard optimization software. In Figure 3, the reconstructions of a parallelogram for different numbers of moments are depicted.

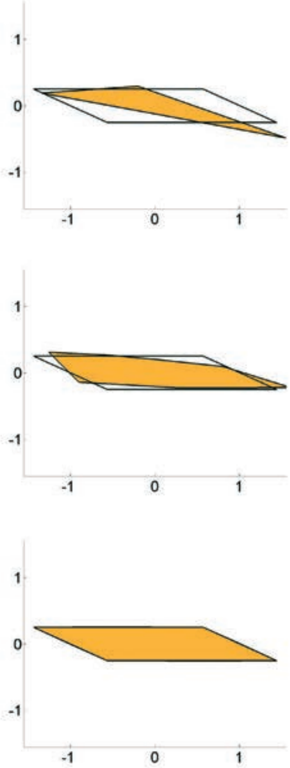

Figure 3:
Reconstruction of a parallelogram from moments of the area measure up to order 0, 2 and 4.

An application of the above mentioned stability result immediately gives the consistency of $\widehat{K}_s$, together with a speed-of-convergence result, in the noise-free case. A consistent algorithm can also be established in the case where the input is distorted by independent Gaussian noise, if the standard deviation decreases appropriately with s. However, the methods require that the measurements of ordinary moments (which are integrals of monomials with respect to $S(K,\cdot)$) are replaced by measurements of integrated spherical harmonics. Very recently, these integrals, also called **harmonic intrinsic volumes**, turned out to be useful in the analysis of non-isotropic Boolean models, see Hörrmann (2014).

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ROTATIONAL INTEGRAL GEOMETRY

In 2014, new local stereological procedures for estimating particle shape and orientation (Ziegel *et al.*, 2014) have been developed, based on recent rotational integral geometry formulae for **Minkowski tensors** (Auneau-Cognacq *et al.*, 2013; Jensen & Ziegel, 2014).

Stereology for tensor-valued functionals has a totally different character than stereology for scalar-valued functionals and requires clever 3D sampling. Furthermore, there are issues of overprojection in thick microscopy sections that need to be taken into account. The new procedures in Ziegel *et al.* (2014) are based on measurements in an **optical slice** centred at a reference point of the particle and use **hitting probabilities** in a correction for non-sampling of the peripheral parts of the particle. In essence, the optical rotator probe is used by means of which overprojection can be avoided. This probe was developed much earlier in Tandrup *et al.* (1997) for the simpler task of estimating particle volume and surface area.

Volume tensors are used in Ziegel *et al.* (2014) for estimating particle shape and orientation. The volume tensor of rank 0 is simply the particle volume, while a normalized version of the volume tensor of rank 1 is the centre of gravity of the particle. The volume tensor of rank 2 contains additional information about particle shape and orientation that can be used to construct **an approximating ellipsoid** with the same volume as the particle, see Figures 1 and 2. As mentioned above, the volume tensors may be estimated by local stereological methods, using the optical rotator design, see the illustration in Figure 3.

For particle processes, the stereological estimators may be combined to provide consistent estimators of the moments of the so-called **particle cover density**. The covariance structure associated with the particle cover density depends on the orientation and shape of the particles. For instance, if the distribution of the typical particle is invariant under rotations, then the covariance matrix is proportional to the identity matrix. A non-parametric test for such isotropy is developed in Ziegel *et al.* (2014). A flexible Lévy-based particle model is proposed, which may be analyzed using a generalized method of moments in which the volume tensors enter.

In Kousholt *et al.* (2014), focus is on Crofton type formulae for translation invariant **surface tensors** where the integration is with respect to the motion invariant measure on lines in n–dimensional space. These formulae are used to construct stereological estimators of such surface tensors based on isotropic random lines, vertical sections and non-isotropic random lines. Finally, the papers Thórisdóttir & Kiderlen (2014) and Thórisdóttir *et al.* (2014) have been published in 2014. The research in these papers was described in Annual Report 2013 where they appeared as *CSGB Research Reports*.

This project **Rotational integral geometry** will be carried over to CSGB II in an extended and further developed form, taking into account and building on the progress that has been made in CSGB I. The new project title in CSGB II is **Valuation theory**. In this extended project, we want to take advantage of the very recent theoretical advances, concerning the algebraic structure (product, convolution, Fourier transform) of tensor valuations (Bernig & Hug, 2014) and locally defined Minkowski tensors (Hug & Schneider, 2014).

Researchers

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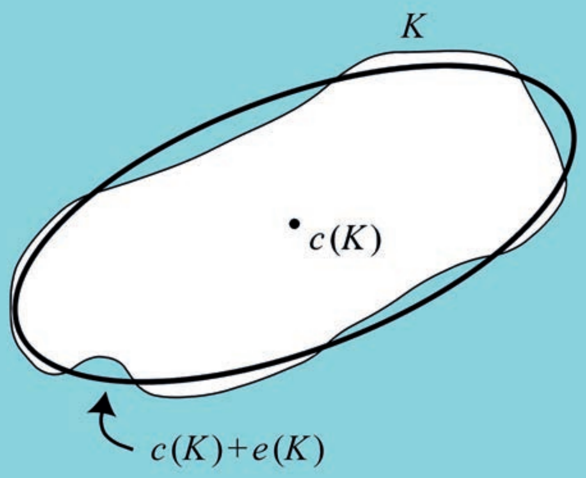

Figure 1:
2D illustration of the ellipsoidal approximation to a particle K. Here, $c(K)$ is the centre of gravity and $e(K)$ is a centred ellipsoid, approximating $K - c(K)$. If K is an ellipsoid, then $K = c(K) + e(K)$.

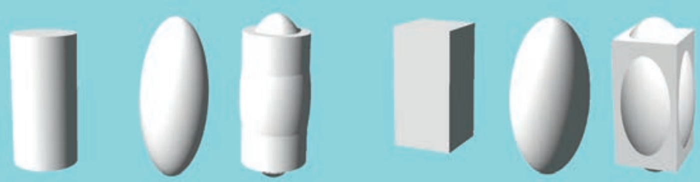

Figure 2:
Ellipsoidal approximations to a right cylinder (left) and a rectangular parallelepiped (right).

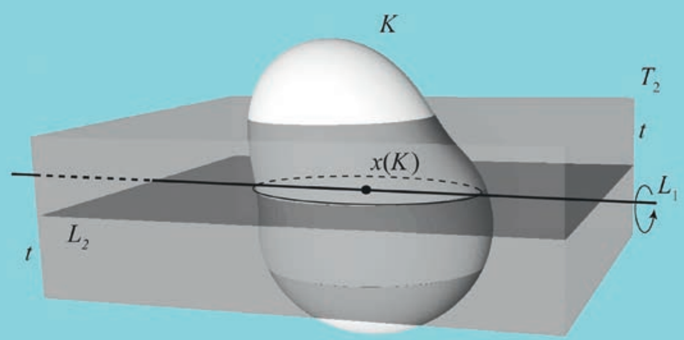

Figure 3:
The optical rotator design by means of which the volume tensors of the particle K may be estimated by local stereological methods. The optical slice T_2 of thickness $2t$, centred at the reference point $x(K)$ of the particle K, is randomly rotated around the 'vertical' axis L_1, that lies in the central plane L_2.

DIGITAL STEREOLOGY

During the first funding period of CSGB, one of the principal findings (Svane, 2014a) in the digital stereology project has been that local algorithms for intrinsic volumes based on binary images are **generally biased**, see also Kampf (2014). More specifically, it has been shown that in dimensions two or higher there exists no local multigrid convergent algorithm for intrinsic volumes except for ordinary volume, based on **binary digital images** of polytopes. Here, multigrid convergence means that the bias of the estimator vanishes asymptotically with increasing resolution.

Since most binary images occur as thresholded grey-value images, the focus has therefore switched to algorithms based directly on **grey-value images** without thresholding. Recent results, now published in Svane (2014b), show the existence of asymptotically unbiased algorithms for surface area and integrated mean curvature based directly on the information available in grey-value images. During 2014, these results have been extended to estimators of **Minkowski tensors** (Svane, 2014c). There do exist fast local digital algorithms to estimate such tensors, based on binary images (Schröder-Turk *et al.*, 2008, 2010). However, the situation seems to be the same here as for estimation of intrinsic volumes where local algorithms based only on the digital information are generally biased.

Surface area and integrated mean curvature can be estimated using $1 \times \cdots \times 1$ configurations whereas larger $k \times \cdots \times k$ configurations are needed in order to gain information about **surface normals**. Moreover, position dependent weights are needed in order to get information about position. This requires a light extension of known results about the asymptotic behaviour of local algorithms. Local estimators of volume, surface and certain mean curvature tensors are then derived in Svane (2014c). The algorithms are asymptotically unbiased, i.e. they converge when the resolution tends to infinity and the **point spread function (PSF)** becomes concentrated near the boundary. In particular, a complete set of estimators are obtained for the Minkowski tensors in 2D. The algorithms require that the PSF is known; at least the knowledge of what a blurred halfspace looks like is required, see Figure 1. Moreover, the resolution has to be sufficiently high compared to the support of the PSF.

As mentioned above, estimators of intrinsic volumes from grey-value digital images are derived in Svane (2014b). These estimators resemble the classical volume estimators as they are also given by lattice point counting, but each lattice point must be weighted according to its grey-value. So far, not much has been known about the precision of these estimators. Even though the mean converges, the **variance** may be large. A low resolution would intuitively result in a large variance. In Svane (2014d), an **asymptotic bound** on the variance is provided when the resolution and PSF changes. It shows that the biggest contribution comes from the resolution. The bound explicitly depends on the algorithm and the underlying PSF. More explicit formulae for the variance are also derived in Svane (2014d) under strong conditions on the underlying set, namely smoothness, convexity and nowhere vanishing Gaussian curvature. See also Figure 2. The variance can be decomposed into a lattice sum depending only on the set through its surface area and an occillating term of at most the same magnitude.

The paper Svane (2014e), concerning local digital estimators of intrinsic volumes for Boolean models, has been published in 2014. This research has been described in CSGB Annual Report 2012.

The digital stereology project will be carried over to CSGB II as the extended project *Digital grey-value stereology* within the work package called **Algorithms**, see page 12-13. Apart from local estimation algorithms on grey-value images, more resource-intense global algorithms will be studied.

Researchers

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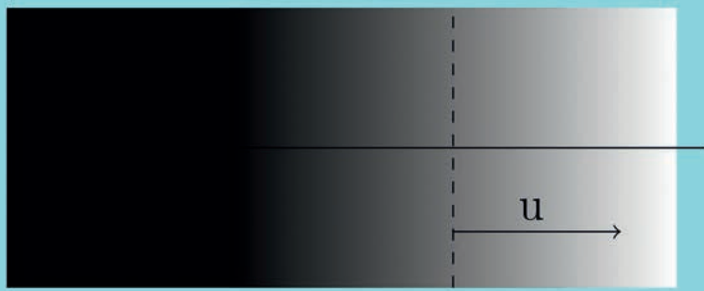

Figure 1:
A grey-scale image of a halfspace, showing the change in grey-values along the horizontal line.

$$\mathrm{Var}(\widehat{S}(f)^{a,a}(X))$$
$$= 2a^{d-1}\omega_d^{-1}\alpha_f^{-2}S(X)$$
$$\times \left(\sum_{\xi \in \mathbb{L}^* \setminus \{0\}} |\mathcal{F}(f \circ \theta^H)(|\xi|)|^2 |\xi|^{-d+1} + Z(a) \right)$$
$$+ o(a^{d-1})$$

Figure 2:
Explicit formula for the variance of a surface area estimator, based on observation in a grey-value digital image. For details, see Svane (2014d, Section 2.4).

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TOPOLOGY AND DIGITAL IMAGE ANALYSIS

When analyzing digital images, e.g. from microscopy, an understanding of the connection between the original object and digital representations of it is essential.

Consider an evenly spaced cubic lattice L with grid-size d on Euclidean three-space $\mathbb{R}^3$. The voxel corresponding to a point $p \in L$ is the cube of side length d centred at p whose edges are parallel to the edges of the lattice cubes. A subset X of $\mathbb{R}^3$ has **digitization** $D_L(X) = X \cap L$ with respect to L, and the **voxel reconstruction** $V_L(X)$ of X with respect to L is the union of the voxels corresponding to all the points of the digitization $D_L(X)$. It is relatively easy to compute invariants of such voxel reconstructions, but it is not clear to what extent these represent the topology or geometry of the original object.

Assume now that the set X is ***r*-regular**, that is, at each point on the boundary ∂X of X, we can find two open tangent balls to ∂X of radius r that are respectively contained in the interior of X and the interior of its complement, see Figure 1. Suppose also that the grid-size d is smaller than $r/\sqrt{3}$. In that case only a rather small list of different configurations of voxels can occur; the list was first obtained by Stelldinger *et al.* (2007). Figure 2 shows an example of such a reconstruction.

The boundary of the digital reconstruction is none the less not usually a manifold-with-boundary, but wedges can always be inserted, in an rather obvious way, giving a **wedged reconstruction** $W_L(X)$, see Figure 3.

We have proved that $W_L(X)$ and X are always homeomorphic, indeed that they are ambient isotopic. This improves on the result of Stelldinger *et al.* (2007), where it is proved, under the same conditions on grid-size and regularity, that a reconstruction homeomorphic to X can be constructed via some rather complicated digital-image-analysis algorithms (amongst them, for example, is a variation of the 'marching cubes' algorithm).

Our proof uses the construction of a nowhere zero vector field on a region between two surfaces diffeomorphic to ∂X. In particular, the vector field is constructed to be transverse to a smoothed version $\partial\widetilde{W}$ of $\partial W_L(X)$. Figure 4 shows the vector field and illustrates its behaviour near the relevant surfaces. Given such a vector field, a corollary of the Poincaré-Hopf theorem shows that ∂X and $\partial\widetilde{W}$ have the same Euler characteristic. Indeed, it follows that these surfaces are homeomorphic - since any closed surface in $\mathbb{R}^3$ is homeomorphic to a sphere with k handles for some k (illustrated in Figure 5), which k being determined by the Euler characteristic. The topological equivalence of ∂X and $\partial W_L(X)$ now follows from the fact that $\partial W_L(X)$ and $\partial\widetilde{W}$ are homeomorphic by construction.

Concluding from this that $W_L(X)$ and X are ambient isotopic requires the application of further results from geometric topology - rather deep results, but well-known.

Researchers

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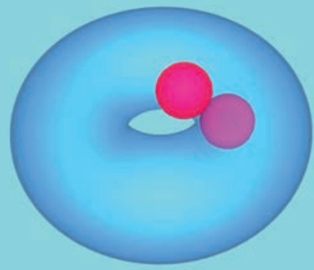

Figure 1:
A subset $X \subset \mathbb{R}^3$ is r-regular if at every point on its boundary there exist two tangent balls of radius r contained in the interior and complement of X, respectively.

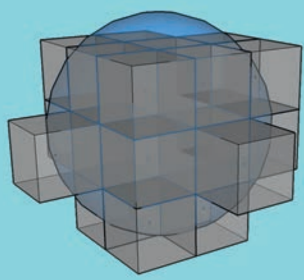

Figure 2:
A digital reconstruction (black cubes) of a ball of radius r (blue) by a cubic lattice of side length $d < r/\sqrt{3}$.

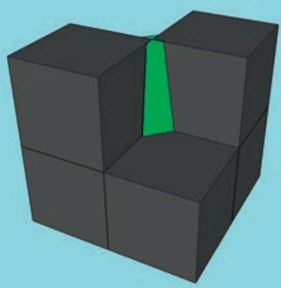

Figure 3:
Wedges, illustrated in green, can be inserted into certain critical edges of the digital reconstruction of an r-regular set to obtain the wedged reconstruction.

Figure 4:
The black arrows illustrate the direction of a nowhere zero vector field near two surfaces diffeomorphic to the boundary of the r-regular set X (orange) and near the boundary $\partial\widetilde{W}$ of a smoothed version of its wedged digital reconstruction $W_L(X)$.

Figure 5:
Spheres with k handles. A sphere with 0 handles is the unit sphere; a sphere with $k > 0$ handles is the k-fold torus."

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SPATIAL AND SPATIO-TEMPORAL POINT PROCESSES

During 2014, a total of six papers on spatial and spatio-temporal point processes have been published or accepted for publications in the **absolute top international statistical journals** (Baddeley *et al.*, 2014; Coueurjolly & Møller, 2014; Deng *et al.*, 2014; Guan *et al.*, 2014; Huang *et al.*, 2014; Lavancier *et al.*, 2014). A detailed description of this research may be found in earlier annual reports.

In Jalilian *et al.* (2014) and Waagepetersen *et al.* (2014), **multivariate point processes** have been studied. Waagepetersen *et al.* (2014) considers the well-known class of multivariate log Gaussian Cox processes and addresses how such models can be used to analyze biologically interesting problems in tropical rain forest ecology. The model involves certain latent random processes, and a cross validation approach to select the number of latent processes is introduced. Jalilian *et al.* (2014) concerns a new class of multivariate so-called product-shot-noise Cox models which allows for flexible modelling of species interactions. Moreover, a project involving the AAU and KU groups has been initiated, concerning inference on shape and interaction characteristics of synaptic vesicles from 3D data, using marked point processes as a framework for the statistical analysis. This research will be continued in CSGB II.

During the process of finishing the book *Analysing Spatial Point Patterns with R* (to appear late 2015, see page 15), Ege Rubak has written a paper together with Adrian Baddeley and Rolf Turner, investigating the analogue of the likelihood ratio test for Gibbs point process models fitted by **maximum pseudolikelihood** (Baddeley *et al.*, 2014). In particular, recent results from the literature are applied to adjust the derived test statistic to obtain a well-known asymptotic distribution. This result extends the possibilities for applying ideas and tools from classical statistics in the statistical analysis (including model selection) for Gibbs point process models.

Mechanistic models for marked point processes have been studied in Møller *et al.* (2014). The paper demonstrates how a random quantitative mark can be associated with a spatio-temporal point process via a conditional intensity function and thereby likelihood inference can be performed. Research concerning spectral representations and models for determinantal point processes on the sphere has been initiated in 2014 and will be continued in CSGB II.

The research on the **mini-column hypothesis** in neuro science, involving a new functional summary statistic, the cylindrical K-function, and a new Poisson line cluster point process model, has been described in Møller *et al.* (2015) and applied in a joint paper (Rafati *et al.*, 2014), involving both the AAU and AU-bio groups. Collaboration with Claudia Redenbach, Universität Kaiserslautern, concerning a comparison between the cylindrical K-function and another popular functional summary statistic, has also been started in 2014.

Wavelets provide representations of images with statistical properties that are much more consistent than e.g. the pixel representation which varies greatly from image to image, and good models and associated inference procedures for wavelet coefficients allow for tasks such as denoising of images. In Møller & Jacobsen (2014), wavelet coefficients have been modelled, using a double stochastic Gaussian-log-Gaussian model. New methods for inference have been introduced - in a frequentist setting, using EM-algorithms, and in a Bayesian setting with the INLA framework. The research has also involved a search for faster algorithms in the interesting special case of changing between Fourier and wavelet bases, using a numerical stable method called generalized sampling, and initial results for unweighted reconstructions (where all measurements are equally important) are promising. Sampling at random locations is also of interest and procedures for generating sampling locations, using point processes, are under development. A future research topic concerns applications of wavelet transformations to the classical summary statistics applied to point processes, such as Ripley's K-function.

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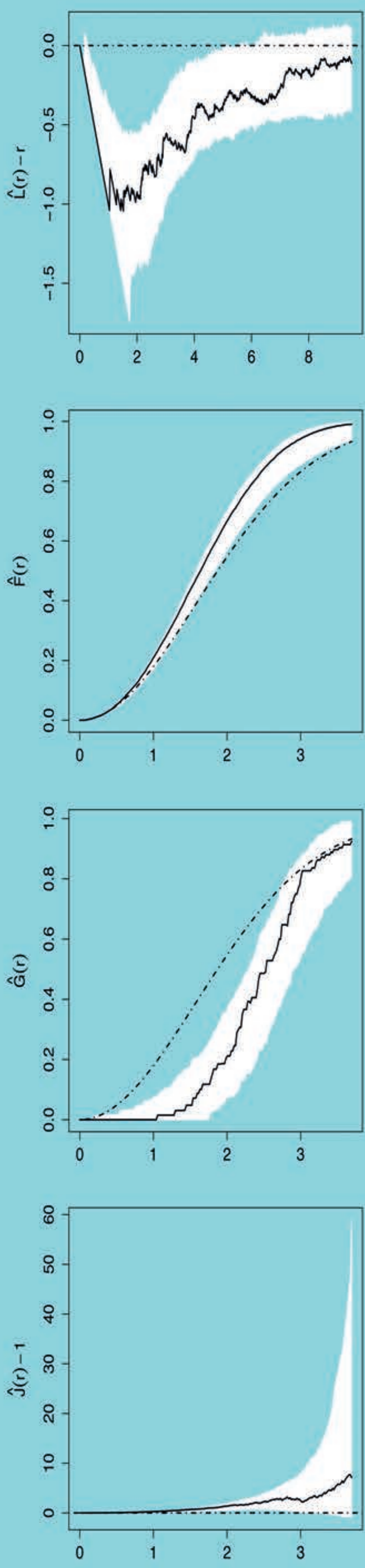

Figure 1:
Non-parametric estimates of functional summary statistics. For details, see Møller *et al.* (2014).

SPACE-TIME LATTICE DATA

One of the main focus points in 2014 for this project has been development of asymptotic theory for **non-Gaussian random fields**.

In Rønn-Nielsen & Jensen (2014), the extremal behaviour of a random field defined as an integral with respect to an infinitely divisible, independently scattered random measure has been investigated. It is shown that the supremum of the field has the same tail asymptotics as the Lévy measure of the underlying random measure, when the tail of the Lévy measure is exponentially decreasing and **convolution equivalent**.

The extremal behaviour of random fields with a convolution equivalent tail of the associated Lévy measure has, in fact, not yet been studied in the literature. Gaussian fields are covered in the monograph Adler & Taylor (2007) while the papers Adler *et al.* (2010, 2013) present results for a **regularly varying tail**. The convolution equivalent distributions have heavier tails than Gaussian distributions and lighter tails than those of regularly varying distributions.

The model class studied in Rønn-Nielsen & Jensen (2014) contains models with **inverse Gaussian** and **normal inverse Gaussian (NIG)** random measure. The extremal behaviour of observations following a non-Gaussian random field model of this type is important e.g. for analysis of brain imaging data (Jónsdóttir *et al.*, 2013). In fact, traditional methods of analysis in brain imaging is based on Gaussian random field theory. These methods may leave small, but significant changes in the signal level undetected, because the assumption of Gaussianity is not fulfilled.

A simulation of one of the random fields studied, a NIG random field, is shown in Figure 1. The tail asymptotics is investigated in Figure 2. For a **NIG random field** in the plane with an exponential kernel function, simulated values of the true tail probabilities $P(\sup_t X_t > x)$ for the supremum of the field within a square is plotted as a function of x (black curve) together with the asymptotic curve (orange). Details concerning parameter choices may be found in Rønn-Nielsen & Jensen (2014). The orange curve approximates the black curve well for x values above 20, say.

It is planned to extend the findings from Rønn-Nielsen & Jensen (2014) to asymptotic results for **excursion sets**. (An excursion set for a random field is the set of points where the corresponding value of the field exceeds a certain level.) We expect to obtain results for the asymptotic size of an excursion set, and will furthermore study the asymptotic behaviour of geometric characteristics such as the number of critical points and the Euler characteristic. It is likely that many techniques from Rønn-Nielsen & Jensen (2014) can be applied in this development.

Another field of study will be to extend the results for the tail asymptotics for the supremum of a field to cases, where the underlying Lévy measure has an exponential tail that is not necessarily convolution equivalent. This will e.g. encompass the important class of models, where the underlying random field follows a **Gamma distribution** – another result that is not covered by the existing literature.

This project will be continued in CSGB II within the new work package **Statistics for stochastic geometry models** that also includes projects on (i) *Monte Carlo envelope tests for random closed sets* and (ii) *Estimation of sample spacing for stochastic processes.* The latter project has developed from a concrete investigation performed in CSGB I, involving CSGB researchers at AU-math, AU-bio and KU, concerning estimation of the thickness of ultra thin sections in electron microscopy, see Sporring *et al.* (2014).

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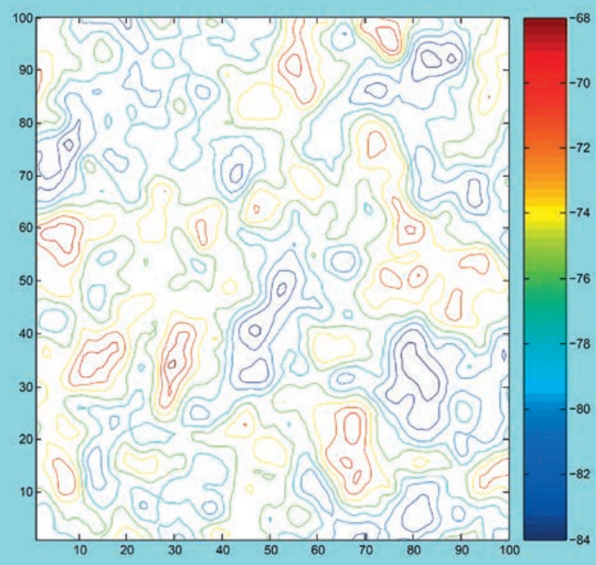

Figure 1:
A simulated normal inverse Gaussian (NIG) random field.

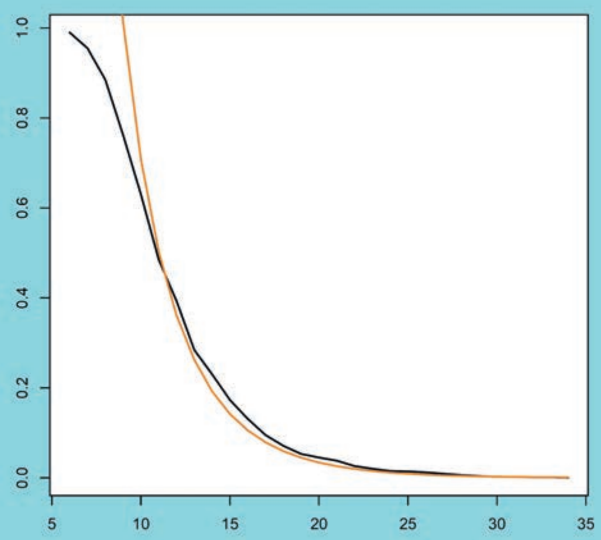

Figure 2: For a NIG random field in the plane with an exponential kernel function, simulated values of the true tail probabilities $P(\sup_t X_t > x)$ for the supremum of the field within a square is plotted as a function of x (black curve) together with the asymptotic curve (orange). Details concerning parameter choices may be found in Rønn-Nielsen & Jensen (2014).

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RANDOM SHAPES

Statistics in non-linear and stratified spaces have been the focus of a large amount of random shapes activities in 2014, including a PhD course on information geometry, multiple workshops, and new theoretical developments.

An entirely new approach to dimensionality reduction on non-linear manifolds was developed with the **diffusion PCA procedure**. In Euclidean space, principal component analysis is widely used both for finding low dimensional approximations of sampled data and for visualization. Generalizing the procedure to non-linear manifolds raises fundamental questions about which properties of the Euclidean procedure to preserve. Based on the Eells-Elworthy-Malliavin construction of Brownian motion, it has been shown how diffusion processes with anisotropic generators can be used to construct a framework for **likelihood estimation** of template and covariance structure for manifold valued data. The procedure avoids the linearization that arises when first estimating a mean or template before performing PCA in the tangent space of the mean. We identified the most probable paths reaching sampled data as extremal paths in the frame bundle of the manifold and used this to derive evolution equations. The paths that generally do not project to geodesics can be used for estimating the likelihood of the model. Efficient numerical implementations was developed by mixing a lifted Riemannian metric and the induced covariance cometric. Collaboration with Henry Jacobs has resulted in further developments in deformation modelling and the LDDMM registration framework (Jacobs & Sommer, 2015; Sommer & Jacobs, 2015). Sommer and Jabobs were both involved in the organization of the **Shape 2014 workshop** at Imperial College in London, see page 14.

Within the **random trees** project, several advances were made on principal component analysis for trees. First, hyperbolic low-distortion embeddings were used for visualization of tree-structured data in a 2-dimensional hyperbolic disk (Amenta *et al.*, 2014). In Feragen *et al.* (2014a), pathological behaviour of geodesic PCA in tree-space was discussed, and in Owen *et al.* (2014), an alternative approach using multiple second principal components in tree-space was presented. An automatic anatomical airway tree labeling algorithm based on geodesic tree-space distances was presented in Feragen *et al.* (2014b); the produced labelings were also used in a clinical study of the effect of inspiration on airway dimensions (Petersen *et al.*, 2014).

In geometric approaches to anatomical brain connectivity networks from **diffusion MRI**, two different shortest path tractography methods were presented: A discrete, graph-based algorithm (Kasenberg *et al.*, 2014) which is flexible in allowing complex diffusion models and constraints, and a continuous algorithm for DTI which models the numerical error in the tractography solution as a Gaussian process (Schober *et al.*, 2014). Both methods are highly applicable for analysis of brain connectivity networks (Cheplygina *et al.*, 2014), and Schober *et al.* (2014) will be the starting point for statistical shape analysis of white-matter fiber bundles. These projects will be continued in CSGB II.

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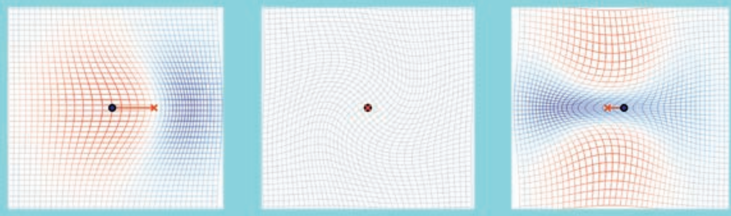

Figure 1:
Examples of deformations of initially square grids. From left to right: 0-th, 1-st and 2-nd. A single jet-particle is located at the blue dots before moving with the flows to the red crosses. Grids are coloured by log-Jacobian determinant. For further details, see Jacobs & Sommer (2015).

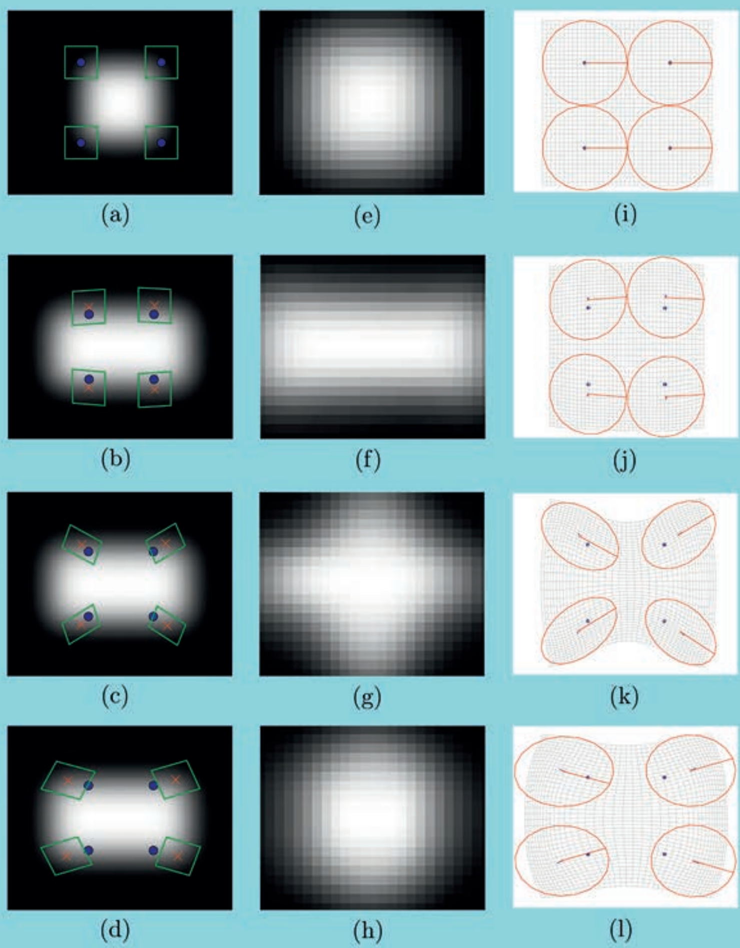

Figure 2:
Matching moving images (b-d) to fixed image (a) using four jet-particles (blue points). Enlarged fixed image and moving images after warping (e-h). Corresponding deformations of an initially square grid (i-l). For further details, see Jacobs & Sommer (2015).

NON-UNIFORM SAMPLING

This project concerns non-uniform sampling of anisotropic or inhomogeneous spatial structures. In 2014, the focus has been on 2D non-uniform systematic sampling and tensor estimation by means of non-uniform random lines.

In Andersen & Ziegel (2014), a new **2D non-uniform systematic sampling** design is introduced that respects the spatial information available. A natural generalization of non-uniform systematic sampling on the line is presented for the case of planar sampling. Under optimal auxiliary information about the function of interest, the resulting estimator based on non-uniform systematic sampling points will have zero variance. However, such optimal auxiliary information requires complete knowledge of the function of interest which is not available in practice. Two alternative practicable choices of non-uniform sampling points are suggested in Andersen & Ziegel (2014). In a simulation study imitating sampling situations in microscopy, the 2D non-uniform systematic designs show in the case of measurement functions for **number estimation** similar efficiency as independent proportional-to-size sampling with replacement. An example of the non-uniform auxiliary information used may be found in Figure 1. This finding is in contrast to the case of **area estimation** where the 2D non-uniform systematic designs are superiour in a number of cases considered, see also Figure 2. In fact, for number estimation in a Poisson process, it is expected that 2D non-uniform systematic sampling does not have higher efficiency than corresponding independent sampling, simply because the numbers counted in disjoint observation windows are independent.

As mentioned in the project **rotational integral geometry**, Crofton type formulae for translation invariant surface tensors have been derived in Kousholt *et al.* (2014). These formulae, dealing with integration with respect to the motion invariant measure on lines, have been used for constructing stereological estimators of surface tensors based on isotropic random lines, vertical sections and **non-uniform random lines**. The latter type of lines is an example of non-uniform sampling. It is shown in Kousholt *et al.* (2014) for a convex body in $\mathbb{R}^2$ that there exists a density of test line directions in an importance-sampling approach that leads to minimal variance of the non-isotropic estimator of one component of the rank 2 tensor, interpreted as a matrix. In practical applications, this density is not accessible, as it depends on the convex body under study. However, there does exist a density independent of the underlying convex body, yielding an estimator with smaller variance than the estimator based on isotropic uniform random lines. If *all* components of the tensor are sought for, the non-isotropic approach requires three test lines, as two of the four components of a rank 2 Minkowski tensor coincide due to symmetry. It should be avoided to use a density suited for estimating one particular component of the tensor to estimate any other component, as this would increase the variance of the estimator. In this situation, however, a smaller variance can be obtained by applying an estimator based on **three isotropic random lines** (each of which can be used for the estimation of *all* components of the tensor).

Models for inhomogeneous point patterns and their statistical analysis have been considered in Prokešová *et al.* (2014). The research on **vanishing auxiliary variables** (Andersen *et al.*, 2014), described in Annual Report 2013, has been accepted for publication in *Scandinavian Journal of Statistics*.

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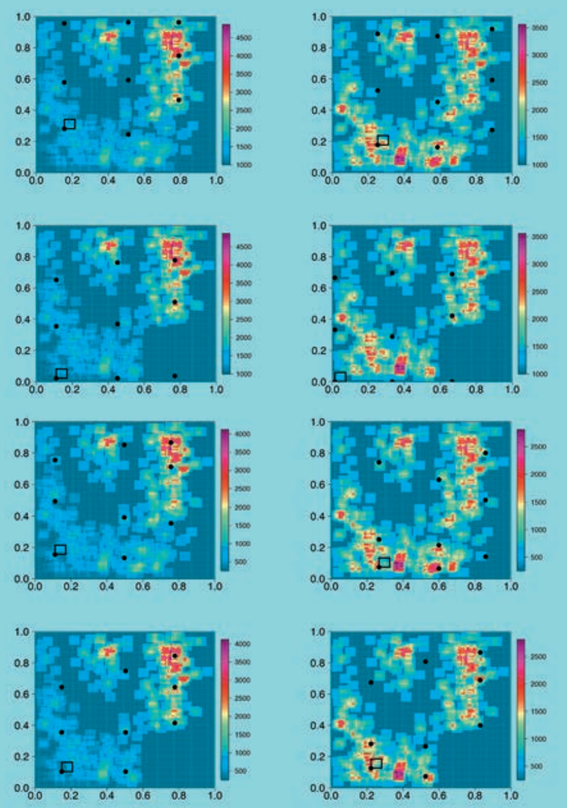

Figure 1:
Eight cases of non-uniform auxiliary information for number estimation, together with one example of sampling points. For more details, see Andersen & Ziegel (2014).

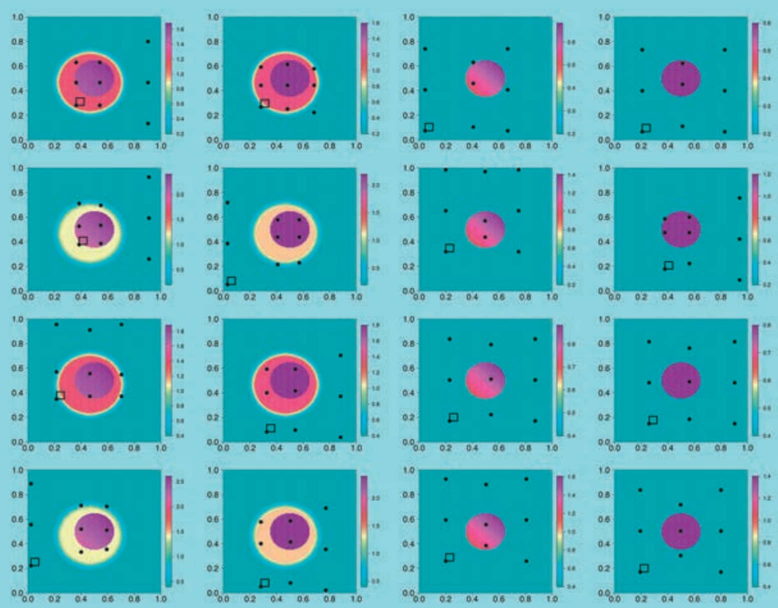

Figure 2:
Sixteen cases of non-uniform auxiliary information for area estimation, together with one example of sampling points. For more details, see Andersen & Ziegel (2014).

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FLUORESCENCE MICROSCOPY TAKEN TO THE MOLECULAR LEVEL

During 2014, we have continued the research on modelling of **cellular protein interactions** using spatial point process models. We have thoroughly assessed the performance of our methodology for Bayesian inference of the spatial distribution of cellular proteins using **FRET data**. In particular, the so-called cross L-function has been used to quantify the nature of spatial interaction in protein configurations generated from the posterior distribution.

The performance of the inference procedure has been assessed in simulation studies using synthetic data generated from **clustered**, **repulsive** and **dimer** (only allowing particular mini-clusters of donor and acceptor points) type of **point patterns**. We have used the Poisson point process as the prior model. From the study, it is clear that good inference results on the cross L-function can be obtained for those cases where the underlying point patterns are not too far from the chosen prior model and for a sufficiently high signal-to-noise ratio (sufficient number of photon counts), see Figure 1.

Further, we have tested the method on an **in vitro empirical data** set of randomly distributed transferrin (iron binding proteins) bound to polylysine slides (Wallrabe *et al.*, 2007). In order to make proper inference on empirical data, it is of great importance to obtain good estimates for the microscope related parameters such as the G- and K-factors (Chen *et al.*, 2006) and for the amplification factors G_D and G_A, related to the photon detector for the donor and acceptor channels. We have derived a simple and efficient method to obtain the parameters G_D and G_A, see Figure 2.

A limiting factor for inferring the spatial distribution of proteins from empirical FRET data is that in general large uncertainty exists concerning the **number of proteins within the sample**. Because a high concentration of donor and acceptor proteins distributed in an unclustered setting can match the channel data just as well as a lower concentration of donor and acceptor proteins distributed in a clustered manner, inference on empirical FRET data remains cumbersome, see Figure 3. We have tried in our synthetic setting to make proper Bayesian inference on the parameter that mainly controls the number of points within a simulation, but noticed that the choice of the prior model heavily influences the outcome.

Our collaboration with Dr. Margarida Barroso from Albany Medical Center in Albany, NY, has continued. Currently, we are studying **a control versus treatment experiment**, which consists of samples of transferrin within cancer cells versus transferrin within normal epithelial cells (Talatia *et al.*, 2014). Results will be presented in Hooghoudt *et al.* (2015). See also Hooghoudt (2015).

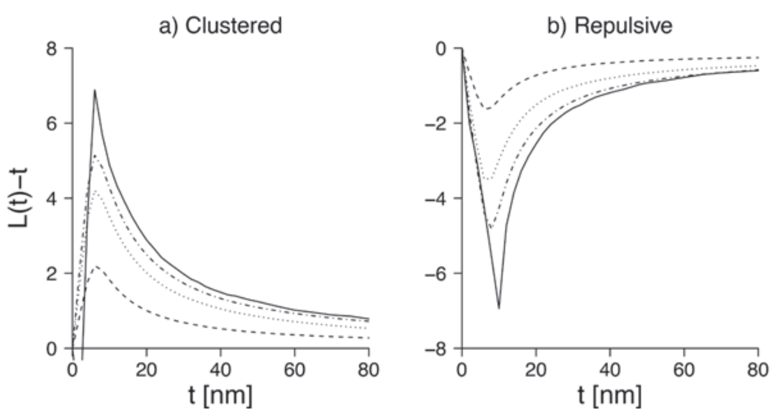

Figure 1:
In each plot, the solid line is the L-function obtained from the synthetic point pattern from which synthetic channel data is generated. The other lines are the posterior mean L-functions, obtained from posterior point pattern realizations for: a low (dashed line), an intermediate (dotted line) and a high (dashed-dotted line) number of photon counts. For moderately clustered and repulsive underlying patterns, good inference results are obtained, see plots a)-b). For more atypical underlying point patterns such as the dimer type (plot c), some aspects of the L-function are not properly inferred.

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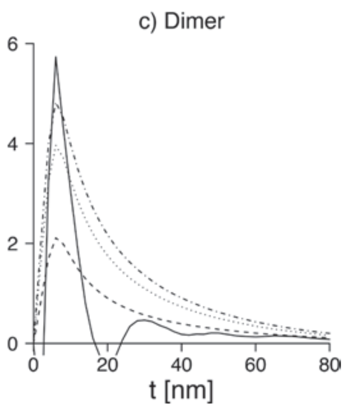

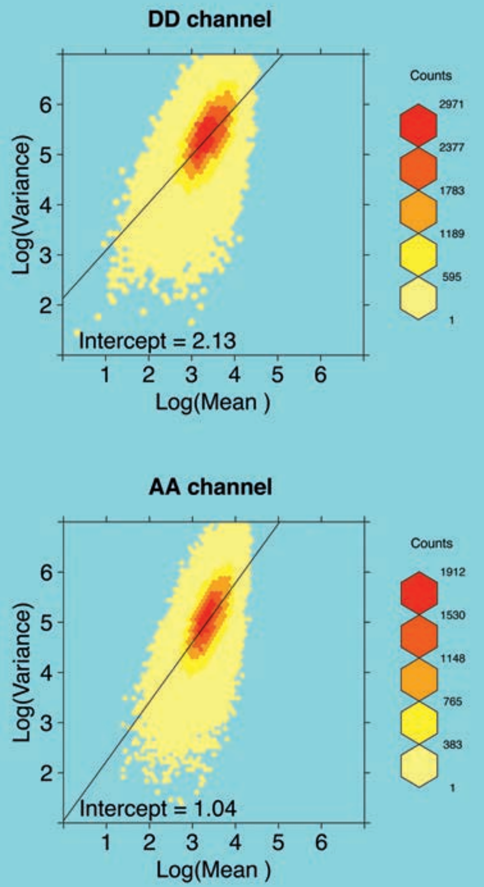

Figure 2:
By remeasuring the FRET channel data of a polylysine sample ten times and plotting for each pixel the intensity variance versus its corresponding pixel intensity mean in a log-log plot, the intercept of the least-squares lines provides a good estimate of the amplification factor in the corresponding channel. From figure: a) $G_D \approx$ exp(2.13), b) $G_A \approx$ exp(1.04).

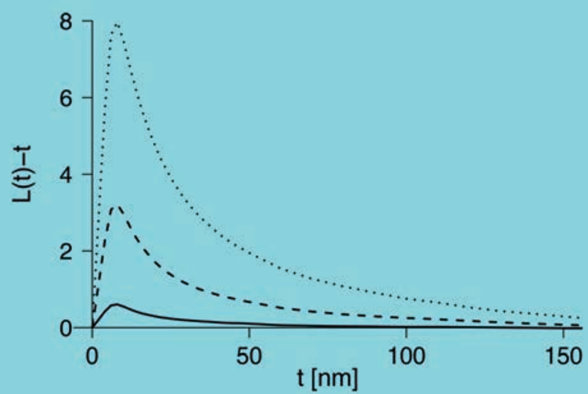

Figure 3:
Posterior mean cross $(L(t) - t)$-function for a polylysine sample. Inference is made for three values of the microscope parameter that controls the signal-to-noise ratio and thereby implicitly the approximate number of points within point pattern realizations from the posterior.
Solid line: #acceptors $\approx 6 \cdot 10^4$ and #donors $\approx 8 \cdot 10^4$ within posterior patterns, dashed line: approximately half the number of points for the solid line case, dotted line: approximately a quarter of the number of points for the solid line case.

MOLECULAR CRYO-EM

Proteins are the central players underlying all processes in living cells. As most cellular processes are too complex for an individual protein, cells have gathered multiple proteins in larger protein complexes often referred to as '**molecular machines**'. To understand the function of protein complexes it is required to observe their structure and changes in structure that accompany e.g. the catalysis of a biochemical reaction or the transmission of communication signals. To this end, we apply **single-particle cryo-EM**, an approach that allows studying the three-dimensional (3D) structure of cellular protein complexes. Notably, cryo-EM often enables the direct visualization of protein complexes even when other methods fail to describe the structure. Particular advantages include the sufficiency of rather small amounts of material as well as the ability to quickly freeze a certain state of the sample, so that unstable or heterogeneous protein complexes can be analyzed by this method.

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Rai, J., Pemmasani, J.K., Voronovsky, A., Jensen, I.S., Manavalan, A., Nyengaard, J.R., Golas, M.M. & Sander, B. (2014): Strep-tag II and Twin-Strep based cassettes for protein tagging by homologous recombination and characterization of endogenous macromolecular assemblies in Saccharomyces cerevisiae. *Mol. Biotechnol.* **56**, 992-1003.

A multi-protein enzymatic complex studied in our group is the **eukaryotic pyruvate dehydrogenase complex** (PDH). PDH is a model of a phosphorylation controlled protein complex that transiently interacts with kinases and phosphatases in order to modulate the biochemical activity according to the actual needs of the cell. We have recently developed a purification strategy that allows studying the native holo-enzyme by single-particle cryo-EM. Our current efforts include the following directions:

(1) We designed yeast strains harboring point mutations of the E1 alpha subunit corresponding to certain mutations found in human, see Figure 1. By this means we hope to be able to clarify how these mutations affect the structural integrity of PDH.
(2) We have previously imaged the PDH particles under negative stain conditions, a specimen preparation method that facilitates to establish the purification method and enables the recording of images that serve as a reference during the further project.

Now we advanced in preparing the sample under native hydrated conditions and could image PDH under cryogenic conditions at the **AU Titan Krios facility**, see Figure 2. Furthermore, we progressed with the expression and purification of a regulatory kinase (Figure 3) as a prerequisite for studying the structure of transient regulatory complexes of PDH. The availability of this protein in pure form will allow us to study the transient interaction of PDH with a regulatory kinase.

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Another multi-protein complex currently under investigation is the **eukaryotic RSC** (Remodels the Structure of Chromatin) complex. RSC is a family member of ATP-dependent chromatin remodeling complexes, which alter the structure and accessibility of the DNA according to a cell's needs and thereby represent key players in gene regulation. Particularly, RSC moves nucleosomes, the basic packaging unit of DNA, along the DNA in order to grant or deny access to the gene for the transcription machinery. We have developed an improved purification method for this complex. Our approach involves **a new genetic split purification tag** that considerably improved the compositional homogeneity of the complexes. The particles could further be shown to be catalytically active and to respond to histone peptides by changes in their structure. Typical two-dimensional projection views of the RSC in its open and closed conformation are shown in Figure 4.

In a collaboration comprising the laboratories of S. Thiel, G.R. Andersen and J.S. Pedersen, we further investigated a **complex of human mannan-binding lectin** (MBL) in complex with the serine protease MASP-1. This protein complex is a part of the innate immune system consisting of a system of serum and cell-bound proteins (the so-called complement system) that counteracts infections by microorganisms. MBL/MASP-1 is a key player in the lectin pathway of the complement system. The projection model deduced from single-particle analysis of MBL/MASP-1 allows conclusions as to possible activation mechanisms of the complex. Particularly, the serine protease domains of MASP-1 protrude from the complex arguing in favor of an inter-complex activation mechanism for this complex, see Figure 5.

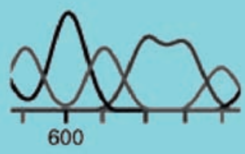


Figure 1:
Sequence analysis of a point mutation introduced in the PDH E1 alpha subunit.

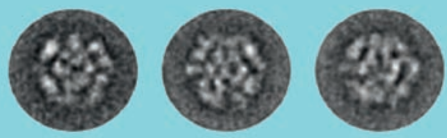


Figure 2:
Native cryogenic class averages upon single-particle image analysis of PDH.

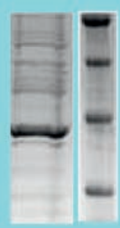


Figure 3:
Expression of a regulatory kinase that can interact with PDH.

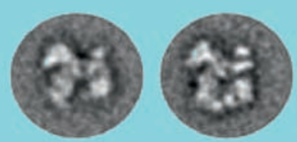


Figure 4:
Representative class averages of RSC in its open (left) and closed (right) conformation.

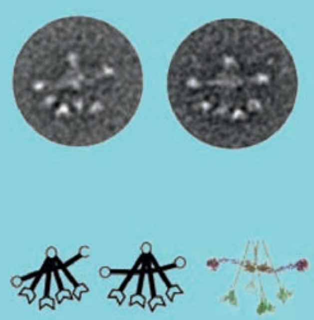


Figure 5:
Projection structures and model of MBL/MASP-1 complexes. Two representative class averages (upper two panels) are shown along with their respective models (lower left two panels). The lower rightmost model depicts a small angle X-ray model of the complex.

CENTRE ACTIVITIES

OVERVIEW

PAST AND PLANNED INTERNATIONAL ACTIVITIES

International conferences and workshops

- *10th French-Danish Workshop on Spatial Statistics and Image Analysis in Biology*
  21 – 23 May 2014, Aalborg
- *Shape 2014 Workshop*
  10 – 13 June 2014, Imperial College, London
- *Stereology Workshop*
  18 – 22 August 2014, Bern
- *FEAST 2014*
  24 August 2014, Stockholm
- *Workshop on Tensor Valuations in Stochastic Geometry and Imaging*
  21 – 26 September 2014, Sandbjerg
- *Oberwolfach Mini-Workshop on Asymptotic Statistics on Stratified Spaces*
  28 September – 4 October 2014, Oberwolfach
- *Stereology Workshop*
  27 – 29 October 2014, University of Washington, Seattle
- *SIMBAD 2015*
  12 – 14 October 2015, Copenhagen

International PhD Courses

- *Stereology Course*
  28 February 2014, Copenhagen
- *International PhD Course on Inverse Problems with Applications in Tomography and Imaging*
  16 – 20 June 2014, Copenhagen
- *Summer School on Deep Learning in Image Analysis*
  18 – 22 August 2014, Rudkøbing
- *Stereology Course*
  16 - 18 September 2014, Sandbjerg
- *PhD Course on Information Geometry in Learning and Optimization*
  22 – 26 September 2014, Copenhagen

The excursion at the *Workshop on Tensor Valuations in Stochastic Geometry and Imaging* went to the oldest town in Denmark, Ribe, and the Wadden Sea. It was a rainful afternoon, but still the participants enjoyed to walk through the narrow streets of Ribe and experience the great views at the Wadden Sea.

WORKSHOP 2014

TENSOR VALUATIONS IN STOCHASTIC GEOMETRY AND IMAGING

21-26 SEPTEMBER 2014, SANDBJERG ESTATE, SØNDERBORG, DENMARK

The workshop was arranged jointly by CSGB and GPSRS, the DFG supported Research Unit Geometry and Physics of Spatial Random Systems.

SCOPE AND STRUCTURE OF THE WORKSHOP

This workshop was dedicated to the mathematical theory and the application of tensor valuations in stochastic geometry and imaging. At the workshop, researchers from stochastic geometry and imaging, who have an interest in the underlying mathematical theory of tensor valuations, were brought together with mathematicians who have an interest in the (potential) application areas of tensor valuations.

Also in recent years, there have been very important advances in the mathematical theory of tensor valuations, for instance, concerning the algebraic structure of tensor valuations and the characterization of local tensor measures. At the same time, tensor valuations are starting to be used in a number of research areas, primarily with the purpose of quantifying the morphology and anisotropy of complex spatial structures.

At the workshop, overview lectures were given by invited experts in the field. The workshop had also shorter research talks.

David Cohen-Steiner

Daniel Hug

Franz Schuster

Rolf Schneider

Semyon Alesker

Gerd Schröder-Turk

Monika Ludwig

Quentin Mérigot

Invited speakers

- Semyon Alesker (Tel Aviv)
- David Cohen-Steiner (INRIA, Sophia Antipolis)
- Daniel Hug (Karlsruhe)
- Monika Ludwig (Wien)
- Quentin Mérigot (Grenoble)
- Rolf Schneider (Freiburg)
- Gerd Schröder-Turk (Erlangen)
- Franz Schuster (Wien)

Springer Lecture Notes in Mathematics

A volume in the Springer Series, Lecture Notes in Mathematics, has been planned based on the invited talks at the workshop and contributions from further experts in the field of valuation theory as well.

CSGB RESEARCH REPORTS 2014

CSGB has its own research report series that mainly publishes mathematical and statistical manuscripts. The major part of these manuscripts will later appear in international journals. The publication traditions are different in computer science and biology for which reason publications by CSGB researchers from these fields usually appear directly in international journals, proceedings, etc.

1. Andersen, I.T., Hahn, U. & Jensen, E.B.V. (2014): Vanishing auxiliary variables in PPS sampling – with applications in microscopy. *CSGB Research Report* **14-01**. To appear in *Scand. J. Stat.*
2. Andersen, I.T. & Ziegel, J.F. (2014): 2D non-uniform sampling. *CSGB Research Report* **14-14**. Submitted.
3. Baddeley, A., Turner, R. & Rubak, E. (2014): Adjusted composite likelihood ratio test for spatial Gibbs point processes. *CSGB Research Report* **14-13**. Submitted.
4. Forbes, P.G.M., Lauritzen, S. & Møller, J. (2014): Fingerprint analysis with marked point processes. *CSGB Research Report* **14-11**. Submitted.
5. Kousholt, A., Kiderlen, M. & Hug, D. (2014): Surface tensor estimation from linear sections. *CSGB Research Report* **14-07**. Submitted. To appear in *Math. Nachr.* (2015) D01:10.1002/mana.201400147.
6. Møller, J., Ghorbani, M. & Rubak, E. (2014): Mechanistic spatio-temporal point process models for marked point processes, with a view to forest stand data. *CSGB Research Report* **14-12**. Submitted.
7. Møller, J. & Jacobsen, R.D. (2014): Gaussian-log-Gaussian wavelet trees, frequentist and Bayesian inference, and statistical signal processing applications. *CSGB Research Report* **14-08**. Submitted.
8. Prokešová, M., Dvořák, J. & Jensen, E.B.V. (2014): Two-step estimation procedures for inhomogeneous shot-noise Cox processes. *CSGB Research Report* **14-02**. Submitted.
9. Rodríguez-Corté, F.J., Ghorbani, M., Mateu, J. & Stoyan, D. (2014): On the expected value and variance for an estimator of the spatio-temporal product density function. *CSGB Research Report* **14-06**.
10. Rønn-Nielsen, A. & Jensen, E.B.V. (2014): Tail asymptotics for the supremum of an infinitely divisible field with convolution equivalent Lévy measure. *CSGB Research Report* **14-09**. To appear in *Adv. Appl. Probab.*
11. Suryaprakash, V., Møller, J. & Fettweis, G.P. (2014): On the modelling and analysis of heterogeneous radio access networks using a Poisson cluster process. *CSGB Research Report* **14-03.** To appear in *IEEE T. Wirel. Commun.*
12. Svane, A.M. (2014): Estimation of Minkowski tensors from digital grey-level images. *CSGB Research Report* **14-04**. Has appeared in *Image Anal. Stereol.* (2015) **34**, 51-61.
13. Svane, A.M. (2014): Asymptotic variance of grey-scale surface area estimators. *CSGB Research Report* **14-05**. Has appeared in *Adv. Appl. Math.* (2015) **62**, 41-73.
14. Ziegel, J.F., Nyengaard, J.R. & Jensen, E.B.V. (2014): Applied tensor stereology. *CSGB Research Report* **14-10**. To appear in *Scand. J. Stat.* (2015) DOI:10.1111/sjos.12138.

PhD Course on Information Geometry in Learning and Optimization

22 - 26 September 2014,
University of Copenhagen, Denmark

	Monday	Tuesday	Wednesday	Thursday	Friday
8	Registration and welcome				
9	Crash course on Differential and Riemannian Geometry 1.1 (Feragen)	Crash course on Differential and Riemannian Geometry 3 (Lauze)	Introduction to Information Geometry 3.1 (Amari)	Information Geometry & Stochastic Optimization 1.1 (Hansen)	Information Geometry & Stochastic Optimization in Discrete Domains 1.1 (Màlago)
10	Crash course on Differential and Riemannian Geometry 1.2 (Feragen)	Tutorial on numerics for Riemannian geometry 1.1 (Sommer)	Introduction to Information Geometry 3.2 (Amari)	Information Geometry & Stochastic Optimization 1.2 (Hansen)	Information Geometry & Stochastic Optimization in Discrete Domains 1.2 (Màlago)
11	Crash course on Differential and Riemannian Geometry 1.3 (Feragen)	Tutorial on numerics for Riemannian geometry 1.2 (Sommer)	Introduction to Information Geometry 3.3 (Amari)	Information Geometry & Stochastic Optimization 1.3 (Hansen)	Information Geometry & Stochastic Optimization in Discrete Domains 1.3 (Màlago)
12	Lunch	Lunch	Lunch	Lunch	Lunch
13	Crash course on Differential and Riemannian Geometry 2.1 (Lauze)	Introduction to Information Geometry 2.1 (Amari)	Information Geometry & Reinforcement Learning 1.1 (Peters)	Information Geometry & Stochastic Optimization 1.4 (Hansen)	Information Geometry & Cognitive Systems 1.1 (Ay)
14	Crash course on Differential and Riemannian Geometry 2.2 (Lauze)	Introduction to Information Geometry 2.2 (Amari)	Information Geometry & Reinforcement Learning 1.2 (Peters)	Information Geometry & Stochastic Optimization 1.5 (Hansen)	Information Geometry & Cognitive Systems 1.2 (Ay)
15	Introduction to Information Geometry 1.1 (Amari)	Introduction to Information Geometry 2.3 (Amari)	Information Geometry & Reinforcement Learning 1.3 (Peters)	Stochastic Optimization in Practice 1.1 (Hansen)	Information Geometry & Cognitive Systems 1.3 (Ay)
16	Introduction to Information Geometry 1.2 (Amari)	Introduction to Information Geometry 2.4 (Amari)	Social activity/ Networking event	Stochastic Optimization in Practice 1.2 (Hansen)	Information Geometry & Cognitive Systems 1.3 (Ay)

A one-week PhD course on information geometry was held at University of Copenhagen, 22 - 26 September 2014. Among others, the course featured a lecture series by **Shun-Ichi Amari**, the founder of information geometry. This led to massive international interest, and we had a waiting list for admitting participants to the course, which had 60 participants from all over the world.

Speakers and lecture titles

- Shun-ichi Amari (RIKEN Brain Science Institute): introduction to information geometry, geometrical structure derived from invariance, applications of information geometry to statistical inference and machine learning
- Nihat Ay (Max Planck Institute for Mathematics in the Sciences and Universität Leipzig): information geometry and cognitive systems
- Nikolaus Hansen (Université Paris-Sud and Inria Saclay – Île-de-France): information geometry and stochastic optimization
- Jan Peters (Technische Universität Darmstadt and Max-Planck Institute for Intelligent Systems): information geometry and reinforcement learning
- Luigi Malagò (Shinshu University, Nagano): information geometry and stochastic optimization in discrete domains
- Aasa Feragen and François Lauze (University of Copenhagen): a crash course on differential and Riemannian geometry

Organization

The course was organized by Aasa Feragen (KU) and Christian Igel (KU). For further information, see the webpage of the course: http://image.diku.dk/MLLab/IG.php.

Baddeley, A.J. , Coeurjolly, J.F., Rubak, E. & Waagepetersen, R. (2014): A logistic regression estimating function for Gibbs point processes. *Biometrika* **101**, 377-392.

Chen, C., Nielsen, M., Karssemeijer, N. & Brandt, S.S. (2014): Breast tissue segmentation from x-ray radiographs. *Phys. Med. Biol.* **59**, 2445-2456.

Cheplygina, V., Loog, M., Tax, D. & Feragen, A. (2014): Network-guided group feature selection for classification of autism spectrum disorder. MICCAI Workshop on Machine Learning for Medical Imaging. *Lecture Notes in Computer Science* **8679**, 190-197. Springer.

Coeurjolly, J.-F. & Møller, J. (2014): Variational approach for spatial point process intensity estimation. *Bernoulli* **20**, 1097-1125.

Deng, C., Waagepetersen, R. & Guan, Y. (2014): A combined estimating function approach for fitting stationary point process models. *Biometrika* **101**, 393-408.

Feragen, A., Nielsen, M., Jensen, E.B.V., du Plessis, A. & Lauze, F.B. (2014): Geometry and statistics: manifolds and stratified spaces. *J. Math. Imaging Vis.* **50**, 1-4.

Greilich, S., Hahn, U., Kiderlen, M., Andersen, C.E. & Bassler, N. (2014): Efficient calculation of local dose distributions for response modeling in proton and heavier ion beams. *Eur. Phys. J. D* **68**, 327.

Hauberg, S., Feragen, A. & Black, M.J. (2014): Grassmann averages for scalable robust PCA. *Conference on Computer Vision and Pattern Recognition (CVPR), 2014 IEEE*, 3810–3817.

Huang, H., Ma, X., Waagepetersen, R., Holford, T., Wang, R., Risch, H., Mueller, L. & Guan, Y. (2014): A new estimation approach for combining epidemiological data from multiple sources. *J. Am. Stat. Assoc.* **109**, 11-23.

Jensen, E.B.V. & Ziegel, J.F. (2014): Local stereology of tensors of convex bodies. *Methodol. Comput. Appl. Prob.* **16**, 263-282.

Karemore, G., Nielsen, M., Karssemeijer, N. & Brandt, S.S. (2014): A method to determine the mammographic regions that show early changes due to the development of breast cancer. *Phys. Med. Biol.* **59**, 6759-6773.

Khanmohammadi, M., Waagepetersen, R.P., Nava, N., Nyengaard, J.R. & Sporring, J. (2014): Analysing the distribution of synaptic vesicles using a spatial point process model. *Proceedings of the 5th ACM Conference on Bioinformatics, Computational Biology, and Health Informatics*, 73-78.

Lin, T.Y., Voronovsky, A., Raabe, M., Urlaub, H., Sander, B. & Golas, M.M. (2014): Dual tagging as an approach to isolate endogenous chromatin remodeling complexes from Saccharomyces cerevisiae. *BBA Proteins and Proteomics* **1854**, 198-208.

Møller, J. & Toftager, H. (2014): Geometric anisotropic spatial point pattern analysis and Cox processes. *Scand. J. Stat.* **41**, 414-435.

Nava, N., Chen, F., Wegener, G., Popoli, M. & Nyengaard, J.R. (2014): A new efficient method for synaptic vesicle quantification reveals differences between medial prefrontal cortex perforated and nonperforated synapses. *J. Comp. Neurol.* **522**, 284-297.

Nava, N., Treccani, G., Liebenberg, N., Chen, F., Popoli, M., Wegener, G. & Nyengaard, J.R. (2014): Chronic desipramine prevents acute stress-induced reorganization of medial prefrontal cortex architecture by blocking glutamate vesicle accumulation and excitatory synapse increase. *Int. J. Neuropsychoph.* **18**, 1-11.

Pai, A., Sommer, S., Darkner, S., Sørensen, L., Sporring, J. & Nielsen, M. (2014): Stepwise inverse consistent Euler's scheme for diffeomorphic image registration. Biomedical image registration: 6th International Workshop. *Lecture Notes in Computer Science* **8545**, 223-230.

Petersen, J., Nielsen, M., Lo, P., Nordenmark, L.H., Pedersen, J.H., Wille, M.M., Dirksen, A. & de Bruijne, M. (2014): Optimal surface segmentation using flow lines to quantify airway abnormalities in chronic obstructive pulmonary disease. *Med. Image Anal.* **18**, 531-541.

Petersen, J., Wille, M.M.W., Rakêt, L.L., Feragen, A., Pedersen, J.H., Nielsen, M., Dirksen, A. & de Bruijne, M. (2014): Effect of inspiration on airway dimensions measured in maximal inspiration CT images of subjects without airflow limitation. *Eur. Radiol.* **24**, 2319-2325.

Puelles, V., Douglas-Denton, R.N., Cullen-McEwen, L., McNamara, B.J., Salih, F., Li, J., Hughson, M.D., Hoy, W.E., Nyengaard, J.R. & Bertram, J.F. (2014): Design-based stereological methods for estimating numbers of glomerular podocytes. *Ann. Anat.* **196**, 48-56.

Rai, J., Pemmasani, J.K., Voronovsky, A., Jensen, I.S., Manavalan, A., Nyengaard, J.R., Golas, M.M. & Sander, B. (2014): Strep-tag II and Twin-Strep based cassettes for protein tagging by homologous recombination and characterization of endogenous macromolecular assemblies in Saccharomyces cerevisiae. *Mol. Biotechnol.* **56**, 992-1003.

Rakêt, L.L., Sommer, S.H. & Markussen, B. (2014): A nonlinear mixed-effects model for simultaneous smoothing and registration of functional data. *Pattern Recogn. Lett.* **38**, 1-7.

Sabers, A., Bertelsen, F.C.B., Scheel-Krüger, J., Nyengaard, J.R & Møller, A. (2014): Long-term valproic acid exposure increases the number of neocortical neurons in the developing rat brain. A possible new animal model of autism. *Neurosci. Lett.* **580**, 12-16.

Schober, M., Kasenburg, N., Feragen, A., Hennig, P. & Hauberg, S. (2014): Probabilistic shortest path tractography in DTI using Gaussian process ODE solvers. Medical Image Computing and Computer Assisted Intervention (MICCAI). *Lecture Notes in Computer Science* **8675**, 265-272. Springer.

Sommer, S.H., Lauze, F.B. & Nielsen, M. (2014): Optimization over geodesics for exact principal geodesic analysis. *Adv. Comput. Math.* **40**, 283-313.

Sporring, J., Khanmohammadi, M., Darkner, S., Nava, N., Nyengaard, J.R. & Jensen, E.B.V. (2014): Estimating the thickness of ultra thin sections for electron microscopy by image statistics. *Proceedings of the 2014 IEEE International Symposium on Biomedical Imaging, Beijing, 29 April – 2 May 2014*, 157-160.

Svane, A.M. (2014a): On multigrid convergence of local algorithms for intrinsic volumes. *J. Math. Imaging Vis.* **49**, 148-172.

Svane, A.M. (2014b): Estimation of intrinsic volumes from digital grey-scale images. J. *Math. Imaging Vis.* **49**, 352-376.

Svane, A.M. (2014c): Local digital estimators of intrinsic volumes for Boolean models and in the design-based setting. *Adv. Appl. Probab.* **46**, 35-58.

Thórisdóttir, Ó. & Kiderlen, M. (2014): The invariator principle in convex geometry. *Adv. Appl. Math.* **58**, 63-87.

Thórisdóttir, Ó., Rafati, A.H. & Kiderlen, M. (2014): Estimating the surface area of non-convex particles from central planar sections. *J. Microsc.* **255**, 49-64.

Treccani, G., Musazzi, L., Perego, C., Milanese, M., Nava, N., Bonifacino, T., Lamanna, J., Malgaroli, A., Drago, F., Racagni, G., Nyengaard, J.R., Wegener, G., Bonanno, G. & Popoli, M. (2014): Stress and corticosterone increase the readily releasable pool of glutamate vesicles in synaptic terminals of prefrontal and frontal cortex. *Mol. Psychiatr.* **19**, 433-443.

CSGB VISITORS - 2014

Sarang Joshi (University of Utah, USA)
1 August 2013 - 31 July 2014

Anders Rønn-Nielsen (University of Copenhagen, Denmark) | 2 – 3, 9 – 14 and 27 – 31 January 2014

Henry Jacobs (Imperial College, London, England)
6 – 10 January 2014

Florian Pausinger (Institute of Science and Technology, Austria) | 13 – 24 January 2014

Emilio Porcu (University Federico Santa Maria, Chile)
9 - 11 February 2014

Daryl Daley (University of Melbourne, Australia)
23 April – 26 May 2014

Viktor Benes (Charles University, Prague, Czech Republic) | 20 – 23 May 2014

Florent Bonneu (Université d'Avignon, France)
20 – 23 May 2014

Line H. Clemmensen (DTU, Denmark)
20 – 23 May 2014

Jean-Francois Coeurjolly (LJK – IRMA Equipe FIGAL, France) | 20 – 23 May 2014

Céline Delmas (INRA, Toulouse, France)
20 – 23 May 2014

Rémy Drouilhet (Laboratoire Jean Kuntzmann, France)
20 – 23 May 2014

Jiří Dvořák (Charles University, Czech Republic)
20 – 23 May 2014

Edith Gabriel (Université d'Avignon, France)
20 – 23 May 2014

Marc G. Genton (King Abdullah University of Science and Technology, Saudi Arabia) | 20 – 23 May 2014

Yongtao Guan (Yale School of Public Health, USA)
20 – 23 May 2014

Gilles Guillot (DTU, Denmark)
20 – 23 May 2014

Katerina Helisova (Czech Technical University in Prague, Czech Republic) | 20 – 23 May 2014

Kiên Kiêu (INRA, Jouy-en-Josas, France)
20 – 23 May 2014

Salme Kärkkäinen (University of Jyväskylä, Finland)
20 – 23 May 2014

Frédéric Lavancier (University of Nantes, France)
20 – 23 May 2014

David Legland (INRA, Jouy-en-Josas, France)
20 – 23 May 2014

Tomáš Mrkvička (University of South Bohemia, Czech Republic) | 20 – 23 May 2014

Nathalie Peyrard (INRA, France)
20 – 23 May 2014

Claudia Redenbach (Technische Universität Kaiserslautern, Germany) | 20 – 23 May 2014

Rachid Senoussi (INRA – Unité BioSp, France)
20 – 23 May 2014

Samuel Soubeyrand (INRA – Unité BioSp, France)
20 – 23 May 2014

Aila Särkkä (Chalmers, Sweden)
20 – 23 May 2014

Alain Trubuil (INRA – Unité BioSp, France)
20 – 23 May 2014

Marie-Colette van Lieshout (CWI, Netherlands)
20 – 23 May 2014

Anna-Kaisa Ylitalo (University of Jyväskylä, Finland)
20 – 23 May 2014

Gennady Samorodnitsky (Cornell University, USA)
1 - 3 June 2014

Anders Rønn-Nielsen (University of Copenhagen, Denmark) | 2 – 3 June 2014

Kenichi Kanatani (Okayama University, Japan)
16 - 20 June 2014

Ville Kolehmainen (University of Eastern Finland)
16 - 20 June 2014

Erkki Somersalo (Case Western Reserve University, USA) | 16 - 20 June 2014

Marion Neumann (Fraunhofer IAIS, Germany)
24 – 25 June 2014

Abd EL-Rahman AL-Absi (University of Aleppo, Syria)
15 August 2014 – 15 February 2015

Dan Claudiu Cireşan (Dalle Molle Institute for Artificial Intelligence, Switzerland) | 17 – 22 August 2014

Aaron Courville (Université de Montréal, Canada)
17 – 22 August 2014

Hugo Larochelle (Université de Sherbrooke, Canada)
17 – 22 August 2014

Ayse Ikinci (Karadeniz Technical University, Turkey)
18 August 2014 – 13 August 2015

Brijnesh Jain (TU Berlin)
29 August 2014

Richard J. Gardner (Western Washington University, USA) | 16 – 26 September 2014

Semyon Alesker (Tel Aviv University, Israel)
21 – 26 September 2014

Shun-ichi Amari (RIKEN Brain Science Institute, Japan)
21 – 26 September 2014

Nihat Ay (MPI for Mathematics in the Sciences and Universität Leipzig, Germany) | 21 – 26 September 2014

Nikolaus Hansen (Université Paris-Sud and Inria Saclay - Île-de-France, France) | 21 – 26 September 2014

Daniel Hug (Karlsruhe Institute of Technology, Germany)
21 – 26 September 2014

Monika Ludwig (TU Wien, Austria)
21 – 26 September 2014

Ilya Molchanov (University of Bern, Switzerland)
21 – 26 September 2014

Florian Pausinger (Institute of Science and Technology, Austria) | 21 – 26 September 2014

Jan Rataj (Charles University, Prague, Czech Republic)
21 – 26 September 2014

Rolf Schneider (Albert-Ludwigs-Universität, Germany)
21 – 26 September 2014

Gerd Schröder-Turk (Institut für Theoretische Physik I, Germany) | 21 – 26 September 2014

Franz Schuster (TU Wien, Austria)
21 – 26 September 2014

Jan Peters (TU Darmstadt and Max-Planck Institute for Intelligent Systems, Germany) | 21 – 27 September 2014

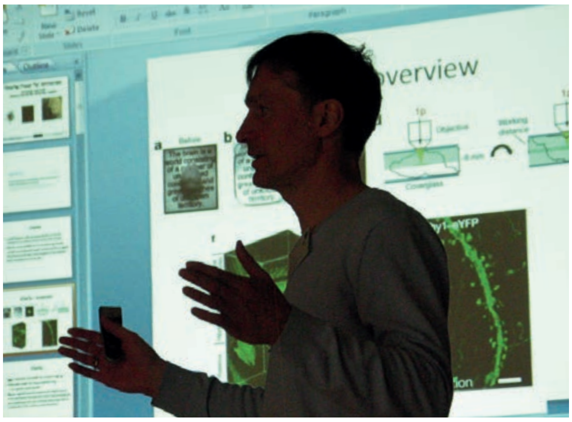
Sébastien Lefèvre (University of Bretagne Sud, France)
November 2014

Anders Rønn-Nielsen (University of Copenhagen, Denmark) | 9 – 14 November and 10 – 12 December 2014

Henry Jacobs (Imperial College, London, England)
9 – 14 December 2014

Eighth Internal CSGB workshop

Sankt Helene, Tisvildeleje, 8 – 9 May 2014



The internal CSGB workshops are held twice a year. They are arranged alternately by the two Aarhus groups (the stochastic geometry and the bioimedical groups), the spatial statistics group at Aalborg University and the image section at University of Copenhagen.




The aim of these internal workshops is to discuss the present status of the CSGB research projects by presentations by the members of CSGB and to plan the further progress of the research projects. Furthermore, new activities arranged by CSGB such as workshops, courses, establishment of new international contacts, etc. are also discussed at these internal workshops. The Eighth Internal CSGB Workshop was arranged by the image section, KU, and took place at Sankt Helene, Tisvildeleje, 8 – 9 May 2014.

An important topic to discuss at the workshop was the new research plan for CSGB II that subsequently was submitted to the Villum Foundation 6 June 2014.

CSGB SEMINARS - 2014

7 January 2014 | Henry Jacobs (Imperial College, London):
Obtaining higher-order spatial accuracy in LDDMM with jet-particles

16 January 2014 | Florian Pausinger (Institute of Science and Technology, Austria):
On persistent intrinsic volumes

10 February 2014 | Emilio Porcu (University Federico Santa Maria, Chile):
Adaptive tapers for space-time geostatistics

2 June 2014 | Gennady Samorodnitsky (Cornell University, New York):
Is the location of the supremum of a stationary process nearly uniformly distributed?

13 June 2014 | Sabrina Tang Christensen (Aarhus):
Surfaces, digitisations and reconstructions

24 June 2014 | Marion Neumann (Fraunhofer IAIS):
Propagation kernels - efficient graph kernels from propagated information

19 August 2014 | Astrid Kousholt (Aarhus):
Surface tensors of convex bodies: integral formulae and uniqueness results

29 August 2014 | Brijnesh Jain (TU Berlin):
Learning on graph orbifolds

11 September 2014 | Anne Marie Svane (Aarhus):
Voronoi-based estimation of Minkowski tensors from digital images

13 October 2014 | Nina Linde Reislev (Danish Research Centre for Magnetic Resonance, Hvidovre Hospital): **The wiring of the blind brain**

13 October 2014 | David Matteson (Cornell University, New York):
A spatio-temporal point process model for ambulance demand

23 October 2014 | Anders Rønn-Nielsen (Department of Mathematical Sciences, University of Copenhagen): **Tail asymptotics for an infinitely divisible field with convolution equivalent Lévy measure**

24 October 2014 | Zhou Hang (École Normale Supérieure de Paris):
Graph reconstruction and verification via distance oracles

5 November 2014 | Sébastien Lefèvre (University of Bretagne Sud):
Multiscale and supervised image analysis in earth observation and remote sensing

11 December 2014 | Henry Jacobs (Imperial College, London):
Higher order accuracy in diffeomorphic image registration: an introduction to jet-groupoids

10th French-Danish Workshop on Spatial Statistics and Image Analysis in Biology

21 - 23 May 2014, Aalborg University, Denmark

The 10th French-Danish Workshop on Spatial Statistics and Image Analysis in Biology took place 21 - 23 May 2014 at Aalborg University. This type of workshop, devoted to spatial statistics and image analysis and their applications in biology (agriculture, ecology, environment, medicine, …), has been arranged alternatively every second year in France and Denmark since 1996. In this workshop, 36 scientists participated, each giving a 25 minutes presentation. A number of these presentations concerned new models and statistical inference procedures for spatial and spatio-temporal point processes.

Ninth Internal CSGB workshop

The Ninth Internal CSGB Workshop was arranged by the stochastic geometry group and took place at Severin Kursuscenter, Middelfart, 6 – 7 November 2014.

Nine research talks were given, the main part of the talks concerned topics within the new research plan for CSGB II. In the late afternoon 6 November 2014, a walk along Lillebælt was arranged.

APPENDIX

CSGB scientific staff

PROFESSORS

- Eva B. Vedel Jensen (EBVJ)
- Christian Igel (CI)
- Jens Ledet Jensen (JLJ)
- Jesper Møller (JM)
- Mads Nielsen (MN)
- Jens R. Nyengaard (JRN)
- Rasmus P. Waagepetersen (RPW)


EBVJ


CI


JLJ


JM


MN


JRN


RPW

ASSOCIATE PROFESSORS

- Johnnie B. Andersen (JBA)
- Sami Brandt (SB)
- Karl-Anton Dorph-Petersen (KADP)
- Aasa Feragen (AF)
- Monika Golas (MG)
- Ute Hahn (UH)
- Asger Hobolth (AH)
- Kristjana Ý. Jónsdóttir (KYJ)
- Markus Kiderlen (MK)
- François Lauze (FL)
- Kim Stenstrup Pedersen (KSP)
- Andrew du Plessis (AP)
- Jakob G. Rasmussen (JGR)
- Björn Sander (BS)
- Jon Sporring (JS)


JBA


SB


KADP


AF


MG


UH


AH


KYJ


MK


FL


AP


JGR


BS


JS


SD


SS

ASSISTANT PROFESSORS

- Sune Darkner (SD)
- Stefan Sommer (SS)


RJ


ML


AR


ER


AMS

POSTDOCS

- Stine Hasselholt (SH)
- Robert Jacobsen (RJ)
- Matthew Liptrot (ML)
- Allan Rasmusson (AR)
- Ege Rubak (ER)
- Anne Marie Svane (AMS)


SFB

SOFTWARE DEVELOPER

- Sine Flarup Budtz (SFB)


ITA


STC


MG


KHJ


JOH


MK


AK


AHR


IR


FS

PH.D. STUDENTS

- Ina Trolle Andersen (ITA)
- Sabrina Tang Christensen (STC)
- Mohammad Ghorbani (MG)
- Katrine Hommelhoff Jensen (KHJ)
- Jan-Otto Hooghoudt (JOH)
- Mahdieh Khanmohammadi (MK)
- Astrid Kousholdt (AK)
- Ali Hoseinpoor Rafati (AHR)
- Jay Rai (JR)
- Farzaneh Safavimanesh (FS)

CENTRE FOR **STOCHASTIC GEOMETRY** AND ADVANCED **BIOIMAGING**

... a very special moment.

Annual Report 2014, published May 2015

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www.math.au.dk

Biomedical group
Danish Neuroscience Centre
Aarhus University, Building 10G, 3rd Floor
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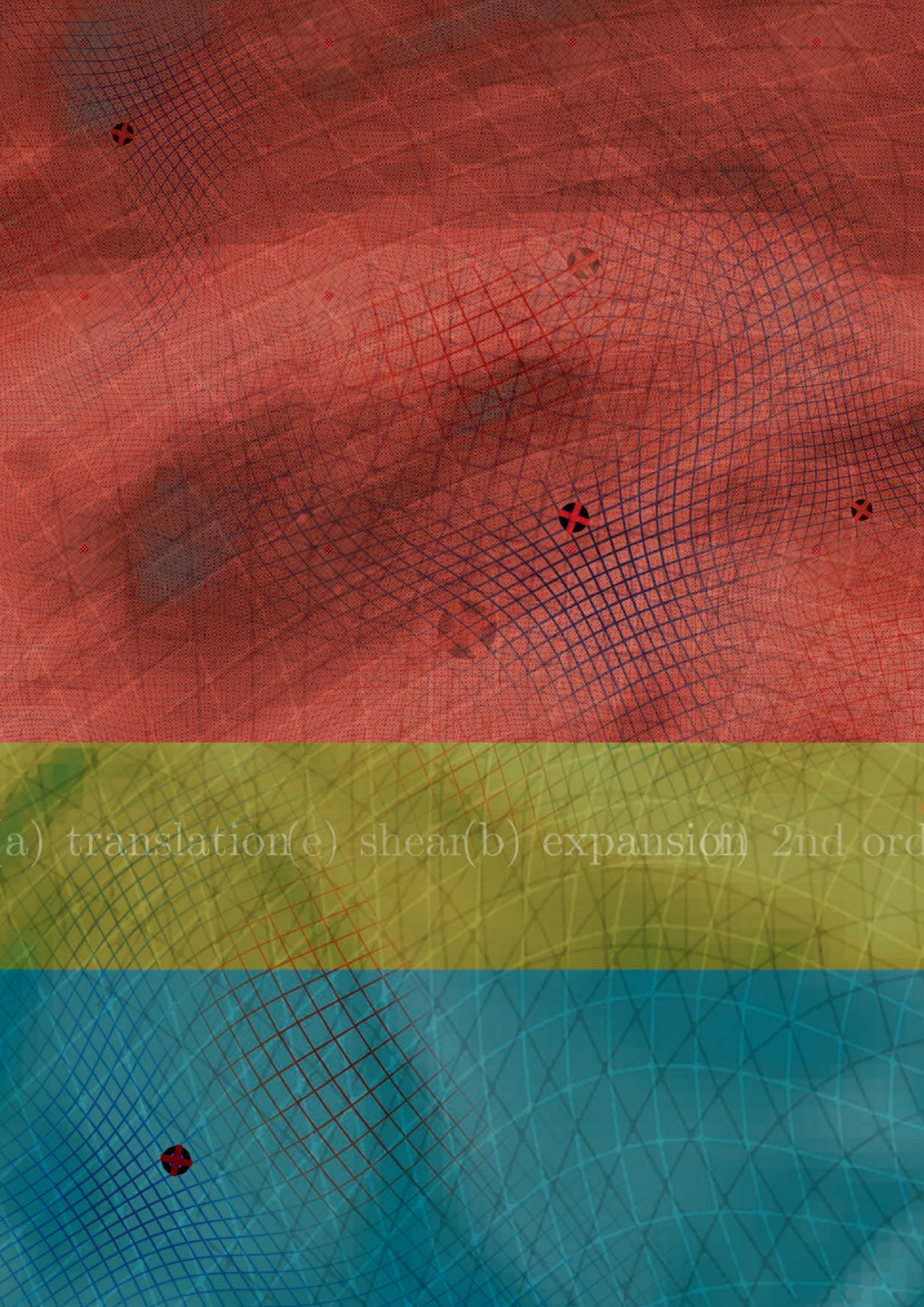
(a) translation
(e) shear
(b) expansion
(f) 2nd ord