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Abstract

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Convergence to Scale Invariant Poisson Processes

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We study weak convergence of a sequence of point processes to a simple point process with particular emphasis on scale invariant point processes as limits. For two deterministic sequences $(z_n)_{n\geq 1}$ and $(s_n)_{n\geq 1}$ increasing to infinity as $n \to \infty$ and a sequence $(X_k)_{k\geq 1}$ of independent integer-valued random variables, we consider the sequence of random measures

$$\nu_n = \sum_{k=1}^{\infty} X_k \delta_{z_k/s_n}$$

and prove that under certain general conditions, it converges vaguely in distribution to the scale invariant Poisson process η on $\mathbb{R}_+$ with intensity $1/x$. An important motivating example relies on choosing $X_k \sim \mathrm{Geom}(1-1/p_k)$ and $z_k = s_k = \log p_k$ where $(p_k)_{k\geq 1}$ is an enumeration of the primes in increasing order. We derive a general result on convergence of the integrals $\int_0^1 x\, d\nu_n$ to the integral $\int_0^1 x\, d\eta$, thus providing a new way of proving certain Dickman convergence results available in the literature.

We then extend our results to multivariate settings and provide sufficient conditions for vague convergence in distribution for a certain broad class of sequences of random vectors that uplift points from the line to the space. This relies on an extension of the convergence results on the line to integrals of functions with unbounded supports.