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Integral Geometry of Fractals: Scaling behavior of Minkowski Functionals

Joint with Philipp Schönhöfer

The shape of a (pre-)fractal $\mathcal{F}$ can be characterized by intrinsic volumes $V_\nu(\mathcal{F})$, which share with the common d-dimensional volume V_d of spatial structures the property of being additive. The scaling behavior [1]

$$V_{d-\nu}(\mathcal{F} \cap \mathcal{W}_\lambda) = v_{\nu;0}\lambda^{d_f} + v_{\nu;1}\lambda^{d_1} + \ldots + v_{\nu;\nu}\lambda^{d_\nu} \; .$$

of these so-called Minkowski functionals in a window $\mathcal{W}_\lambda$ of size λ can be derived for iterated pre-fractals by standard techniques of integral geometry yielding a family of fractal dimensions d_ν. The subdimensions $d_\nu < d_f$ are determined by fractal dimensions of 'hidden' sub-fractals which are produced by suitable cuts through the structure. The shape of fractals can be well characterized by a complete set of scaling amplitudes $v_{\nu;\mu}$. Thus, additive measures generalize naturally the concept of fractal dimension beyond the scaling of the 'content' of a spatial structure. We illustrate the method by applying it on iterated fractals (Sierpiński carpet, Menger's sponge), random iterated fractals (Mandelbrot percolation) and random fractals (diffusion limited aggregation).

We study also the effects of anisotropy on the scaling behavior beyond the fractal dimension by applying tensorial functionals [2]. It can be shown that Minkowski tensors of anisotropic pre-fractals scale with additional subdimensions. In addition, for anisotropic pre-fractals even scalar Minkowski functionals exhibit multiple edge subterms which merge for the isotropic case.

[1] K. Mecke, *Additivity, Convexity, and Beyond: Applications of Minkowski Functionals in Statistical Physics*, Lecture Notes in Physics **554**, 111–184 (2000).

[2] Philipp Schönhöfer and Klaus Mecke, *The Shape of Anisotropic Fractals: Scaling of Minkowski Functionals pages*, 39-52 in: *Progress in Probability, Vol. 70: Fractal Geometry and Stochastics V*, edited by Christoph Bandt, Kenneth J. Falconer, and Martina Zähle, (Birkäuser, Springer, 2015).